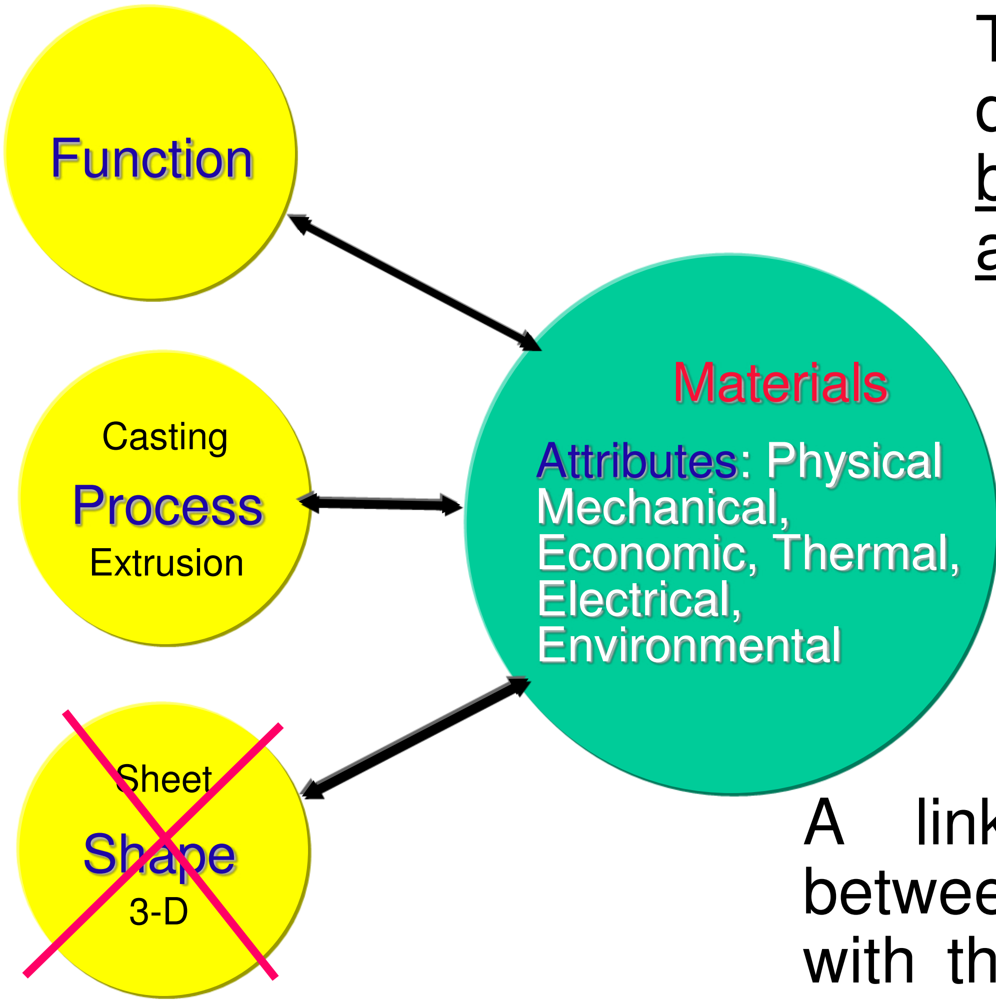
Materials Performance Indices

(Without Shape)

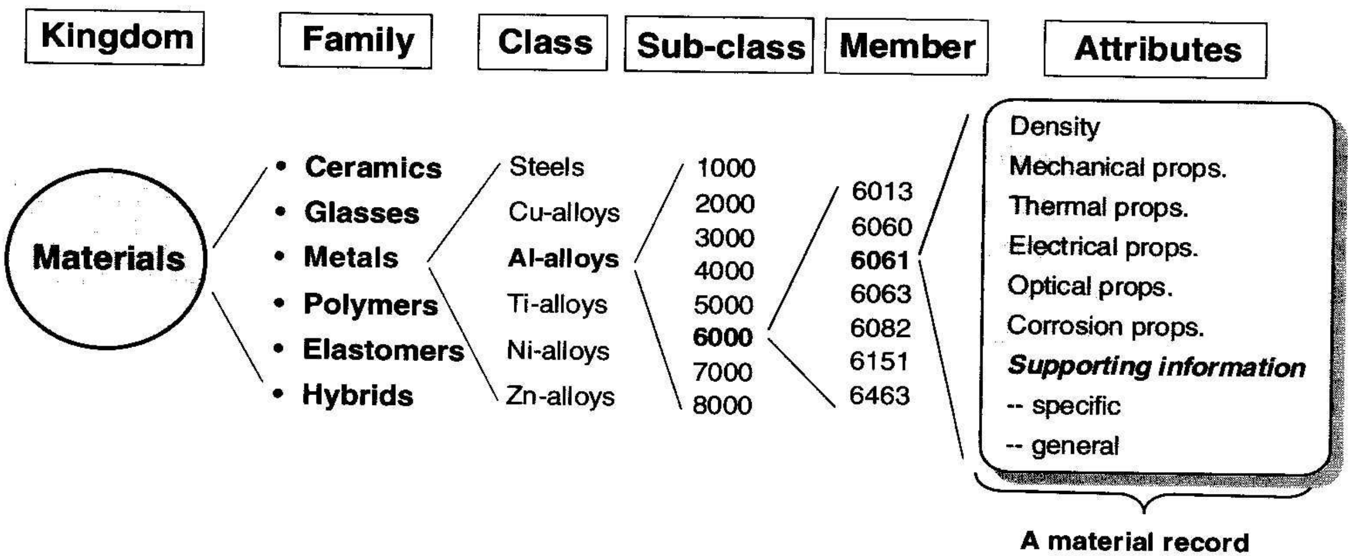


Adapted from M.F. Ashby

The selection of a material depends on the <u>interaction between the MATERIAL and FUNCTION.</u>

A link must be established between MATERIAL, FUNCTION with the PROCESS and SHAPE playing an important role

MATERIAL ATTRIBUTES



The taxonomy of the kingdom of materials and their attributes. Computer-based selection software stores data in a hierarchical structure like this.

- An engineering component has:

  (boundary condition for Materials Selection)

1. Function: to carry load, transmit heat, contain a pressure, etc..

   What does the component do?

2. Objectives: as cheap as possible, light, safe, strong, etc…

   What is to be Maximised or Minimised?

3. Constraints: subject to constraints such as carry load without failure, certain dimensions are fixed, cost is within limits etc…

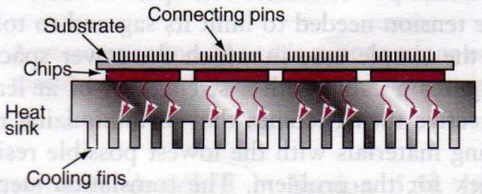
   - *What non-negotiable conditions are to be met? (Rigid)*
   - *What negotiable but desirable conditions? (Soft)*

4. Free Variables: materials choice, cross-section area, thickness, and length are free

Which design variables are free? (variables which can be changed)

e.g. an engineering component has:
(boundary condition for Materials Selection)

Heat Sink for hot microchips



A heat sink for power micro-electronics. The material must insulate electrically, but conduct heat as well as possible.

Function	Heat sink
Objective	Maximize thermal conductivity
Constraints	- Materials must be good insulator - All dimensions are specified
Free variables	Choice of material

- Two concepts are used in the selection procedure:

1. **Materials Performance Index**

   Combination of materials properties that characterise the performance of a material in a given application (Ashby)

2. **Materials Selection Charts**

   Plots of materials properties that form the maximising factors

Two concepts are used in the selection procedure:

1) Material Performance Indices

- Performance of a component/structure is specified by:
  1. *Functional requirements (function) (F) e.g. carry loads, transmit energy, store energy etc.*
  2. *Geometry, (G)*
  3. *Materials properties, (M)*

Performance:

P = f [(functional requirements, F); (geometry, G); (materials properties, M)]

$$P = f\left(F, G, M\right)$$

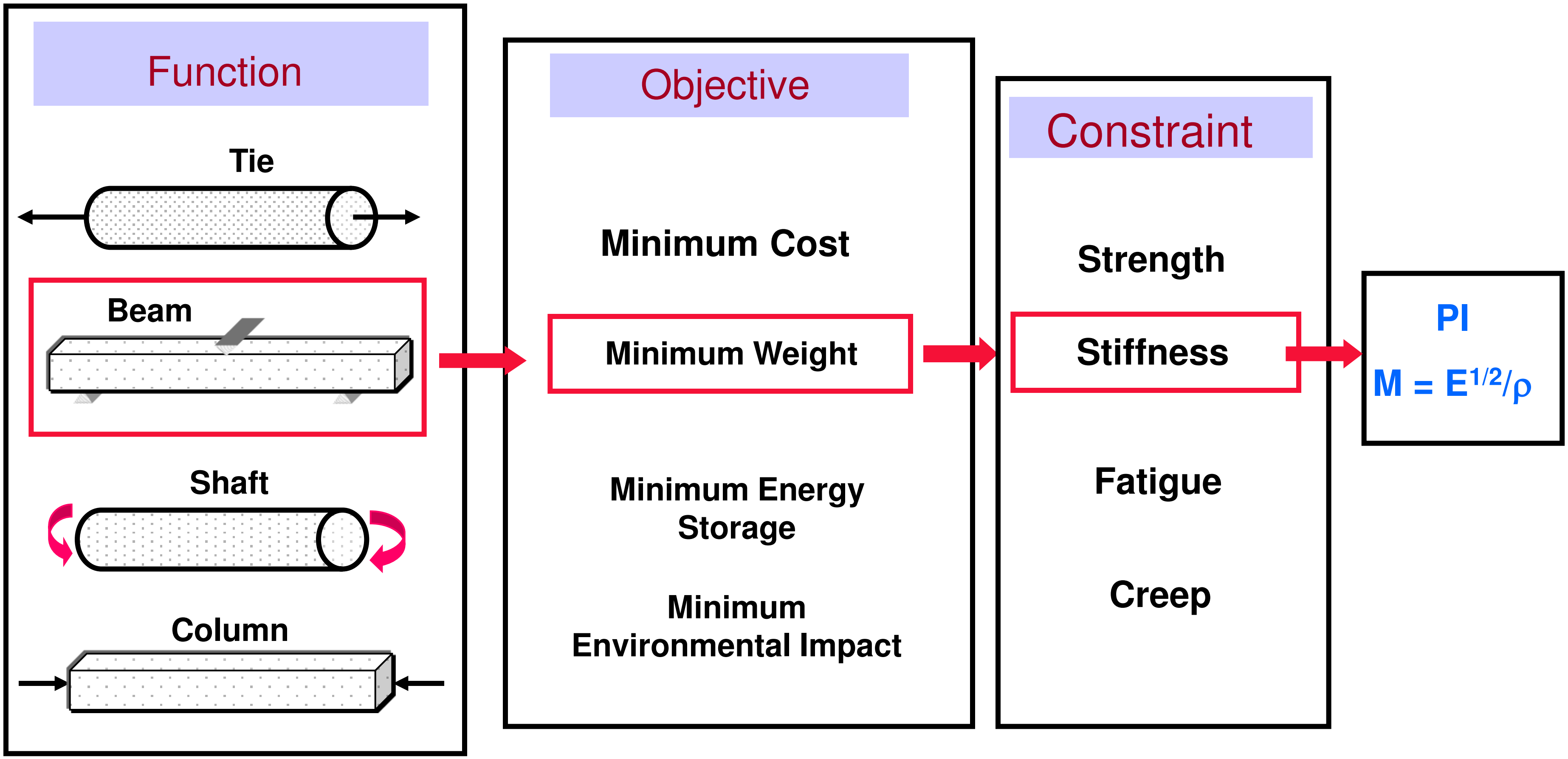
- Optimum design is selection of a material which maximise (strength, stiffness) or minimise (weight, cost) the performance, P.

- In many cases, the function, geometry and materials properties are independent of each other and are said to be separable.

$$P = f_1(F) \times f_2(G) \times f_3(M)$$

- Which means that the optimum selection of a material is independent of the details of the design. It is the same for all geometries and values of functional requirements.

- Performance is maximised by maximising $f_3(M)$ and is called "*Performance Index, M*"



Specification of *Function*, *Objective* and *Constraint* leads to a material performance index M. (after Ashby, 1999)

Functions and Constraints	Maximise (Stiffness)
Tie (tensile strut)	
Stiffness, length specified, section area free	E/ρ
Beam (loaded in bending)	
Stiffness, length, shape specified, section area free	$E^{1/2}/\rho$
Stiffness, length, height specified, width free	E/ρ
Stiffness, length, width specified, height free	$E^{1/3}/\rho$
Panel (flat plate, loaded in bending)	
Stiffness, length, width specified, thickness free	$E^{1/3}/\rho$
Panel (flat plate, buckling failure)	
Collapse load, length and width specified, thickness free	$E^{1/3}/\rho$

After M.F. Ashby

Strength (not stiffness)

Functions and Constraints	Maximise (Strength)
Tie (tensile strut)	
Stiffness, length specified, section area free	σ_f / ρ
Beam (loaded in bending)	
Stiffness, length, shape specified, section area free	$\sigma_f^{2/3} / \rho$
Stiffness, length, height specified, width free	σ_f / ρ
Stiffness, length, width specified, height free	$\sigma_f^{1/2} / \rho$
Panel (flat plate, loaded in bending)	
Stiffness, length, width specified, thickness free	$\sigma_f^{1/2} / \rho$
Panel (flat plate, buckling failure)	
Collapse load, length and width specified, thickness free	$\sigma_f^{1/2} / \rho$

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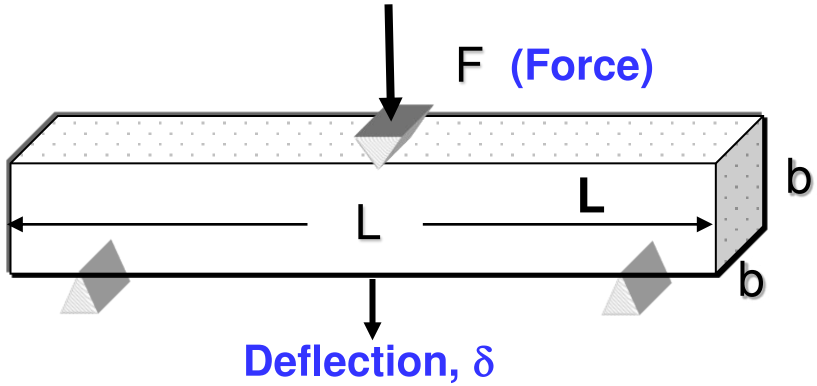
Example 1: Performance Index for a light, stiff beam



- FUNCTION : Beam (bending)
- OBJECTIVE : minimise mass (weight)
- CONSTRAINT(S): Stiffness is $S = F/\delta$ (must not deflect more than δ)
- FREE VRIABLE(S): Cross-sectional area (A), material

- The beam of square section, loaded in bending. Its stiffness is S = F/δ (must not deflect more than δ)

- Constraint on stiffness is given by:

$$S = \frac{F}{\delta} \geq \frac{C_1 E I}{l^3} \qquad (1)$$

- For a square cross section, "*I*" is given by:

$$I = \frac{b^4}{12} = \frac{A^2}{12} \qquad (2)$$

- The mass (Objective), m, is given by:

$$m = Al\rho \qquad (3)$$

- The free variable is the cross section area, $A = b^2$
- Combining equations (1), (2) and (3), the free variable, A, can be eliminated and we then obtain:

$$m \geq \left(\frac{12S}{C_1 l}\right)^{1/2} l^3 \left(\frac{\rho}{E^{1/2}}\right) \tag{4}$$

F G M → PERFORMANCE INDEX

The best materials for a light, stiff beam are those with the smallest value of $\rho/E^{1/2}$. Therefore, to minimize the mass

$$M = \frac{E^{1/2}}{\rho}$$

PERFORMANCE INDEX

OBJECTIVE : minimize mass (weight)

Example 1: Performance Index for a light, stiff beam

- A light and stiff beam (*cross section free*) is one with the largest: $$M = \frac{E^{1/2}}{\rho}$$

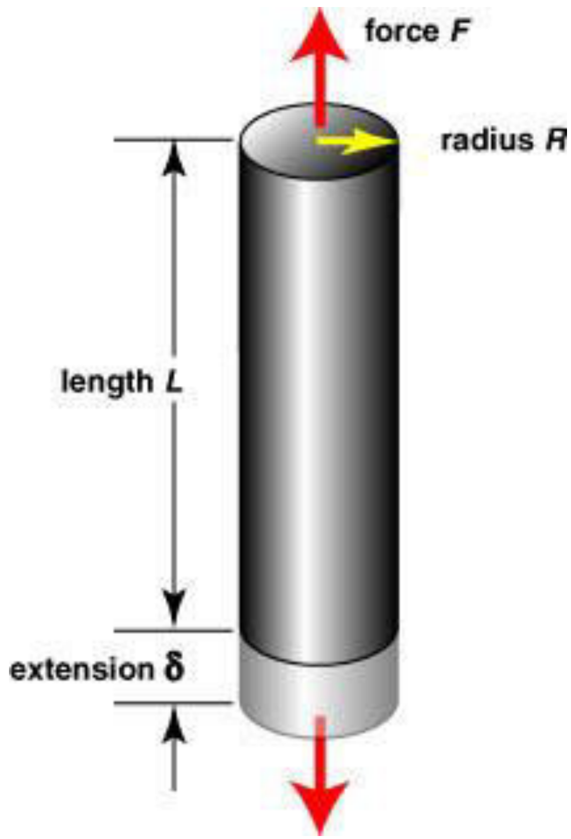
- If *height is free*, the material index is: $$M = \frac{E^{1/3}}{\rho}$$

- If the *width is free*, the material index is: $$M = \frac{E}{\rho}$$

Steps in deriving a “performance Index”

1. Identify the primary FUNCTION
2. Develop an equation for the OBJECTIVE (objective function): e.g; weight, cost, etc… (to be maximised or minimised). Objective function contains one or more free variables.
3. Identify the CONSTRAINTS (which must be met), rank them in order of importance
4. Identify the FREE VARIABLES (unspecified)
5. Develop equations for the constraints (no failure, no buckling, cost below target…)
6. Eliminate the free variable(s) in the objective equation using the constraints.
7. Group the variables into 3 groups: F, G, M
8. Read-off the grouping of materials properties, (called the “PERFORMANCE INDEX”), which maximise the objective

Example 2: Performance Index for a light and stiff cylindrical tie



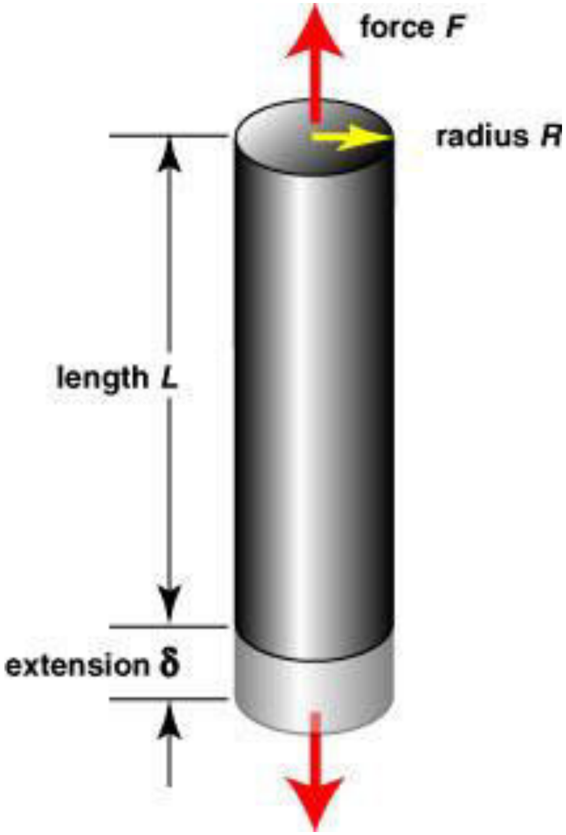
- A tie is loaded in tension (FUNCTION) (stiffness limited design at lowest mass)
- Specified dimensions (CONSTRAINTS): *F*, *L* and δ.
- Free variable: cross-sectional area, *A*.

- Mass (OBJECTIVE):

$$m = Al\rho \qquad (1)$$

- Deflection:

$$\delta = L\varepsilon = L\frac{\sigma}{E} = L\frac{F}{AE} \qquad (2)$$



- This gives the area A;

$$A = \frac{FL}{\delta E} \qquad (3)$$

- Substituting (3) into (1) gives:

$$m = \frac{FL}{E\delta} L\rho = \frac{FL^2}{\delta} \frac{\rho}{E} \qquad (4)$$

- To minimise the weight (mass) the material performance index

$$M = \frac{E}{\rho} \qquad (5)$$

should be maximised and the E - ρ chart will be used to select the optimum material

Example 3: Performance Index for a light & strong cylindrical tie

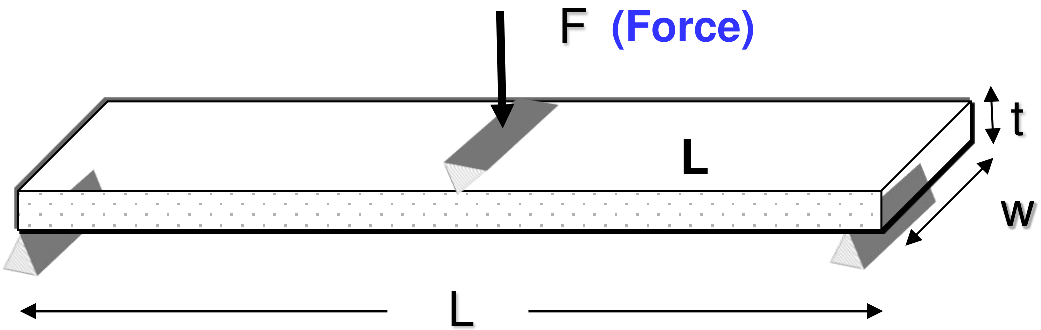
- A tie is loaded in tension (FUNCTION), carry load, F without failure
- Specified dimensions (CONSTRAINTS): *F, L, failure* (σ_f)
- FREE VARIABLE: cross-sectional area, *A.*
- Reduce Mass (OBJECTIVE):

$$m = Al\rho \qquad (1)$$

- The tie must carry load F without failure: $\sigma_f = \dfrac{F}{A}$ (2)

$$m = \frac{F}{\sigma_f} l\rho \quad \Rightarrow \quad M = \frac{\sigma_f}{\rho}$$

Example 4: Performance Index for a light and stiff panel (fixed width)



FUNCTION	Panel (beam): bending
OBJECTIVE	Reduce weight: $m = AL\rho = w\ t\ L\ \rho$ (1)
CONSTRAINT	Stiffness $S = F/\ \delta = CEI/\ L^3$, $I = w\ t^3/\ 12$ (2)
FREE VARIABLE	Thickness, t

We get the free variable, t, from Eq. (2),

$$S = \frac{CEI}{L^3} = \frac{CEwt^3}{12L^3} \Rightarrow t = \left(\frac{12SL^3}{CEw}\right)^{1/3}$$

Replacing t into Eq. 1, we get:

$$m = \left(\frac{12Sw^2}{C}\right)^{1/3} L^2 \left(\frac{\rho}{E^{1/3}}\right)$$

We select materials with the largest

$$M = \frac{E^{1/3}}{\rho}$$

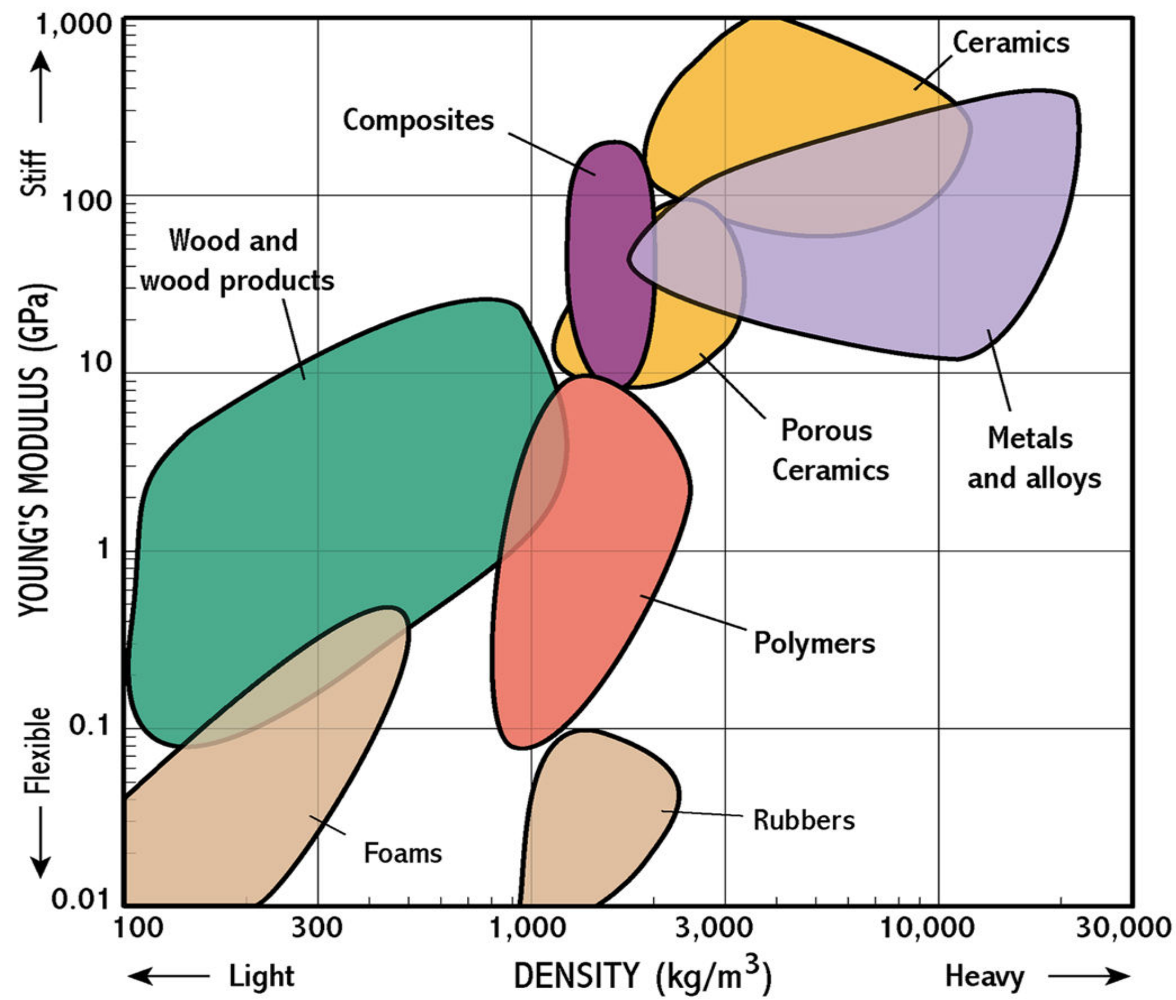
Two concepts are used in the selection procedure:

2) Materials Selection Charts

- The selection of the optimum material is made more simple by the use of "*Materials Selection Charts*"

- Materials selection charts are plots of the properties that form the maximising factors.

- The performance index is made up of 2 properties (e.g; E & ρ). The material selection chart is then created with axis (log by default) of E and ρ.



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- The performance index, M, plots as a diagonal line on the chart. Its slope is very important.

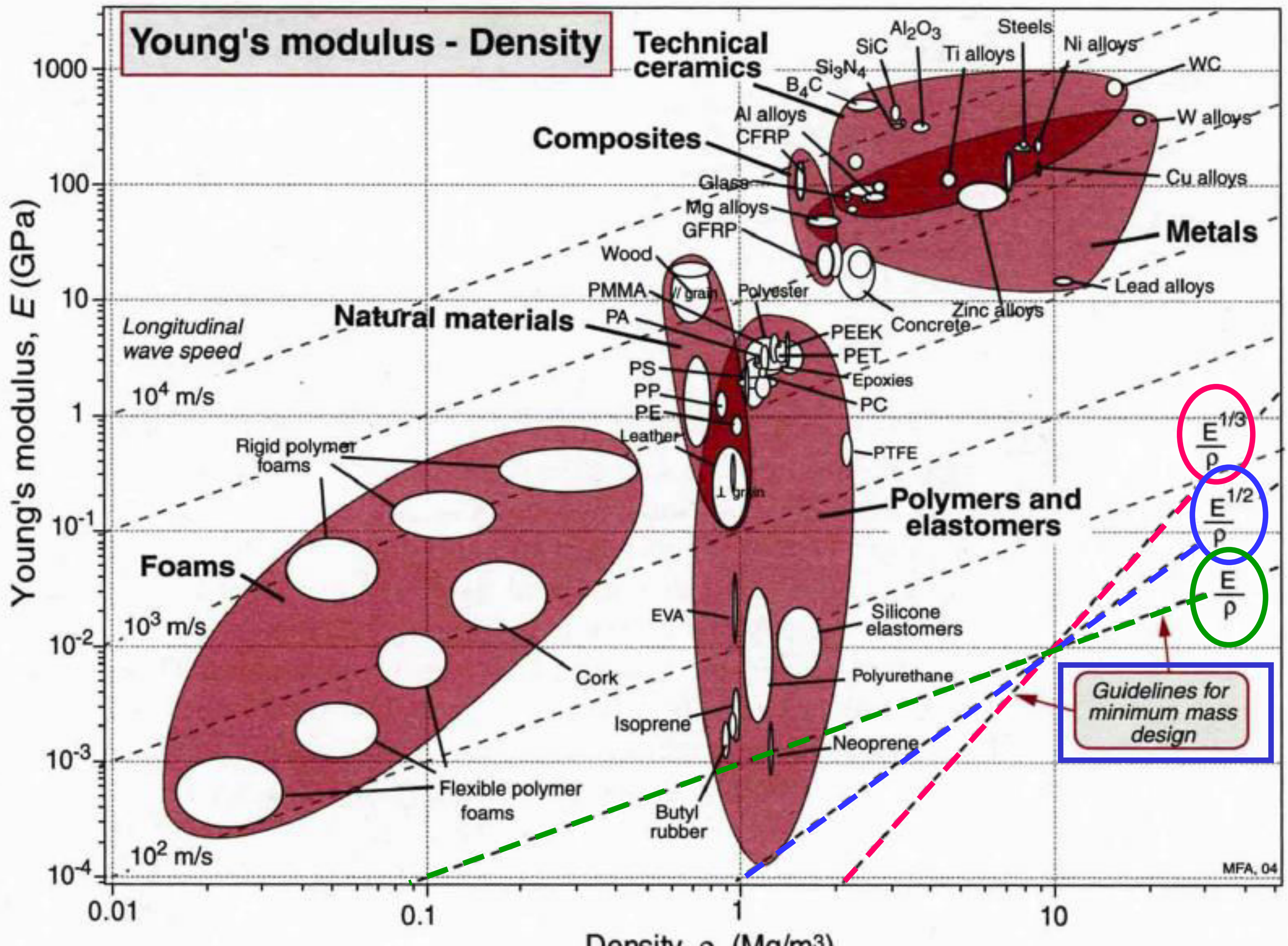
- For the performance index $M = E/\rho$. We take log;

$$\text{Log } E = \log M + \log \rho$$

- A line of slope 1 on the chart describes the index; its position is determined by the value of M.

- For $M = E^{1/2}/\rho$ and $E^{1/3}/\rho$, gives lines with slope 2 and 3 respectively.

- They are called "*design guidelines*"



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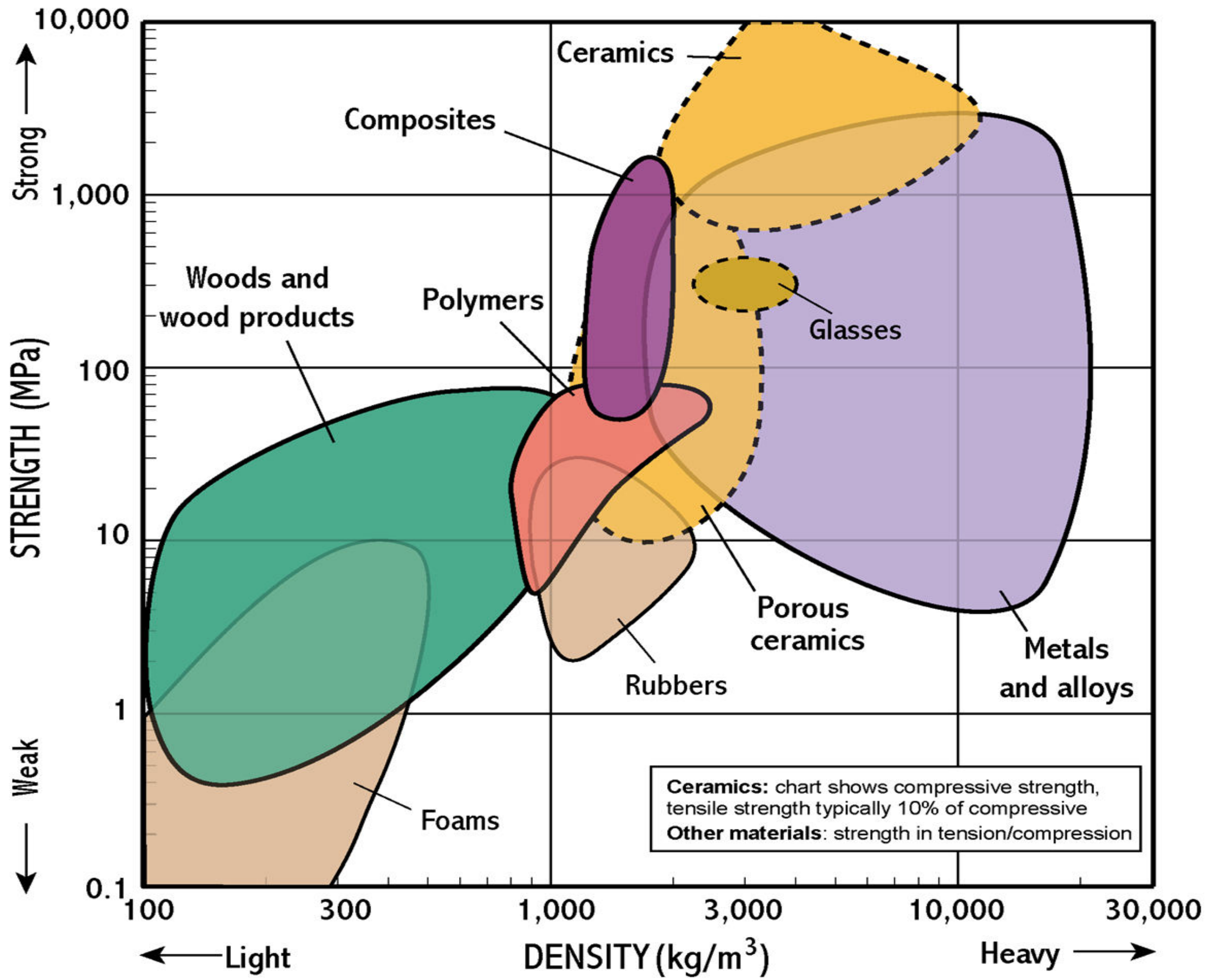

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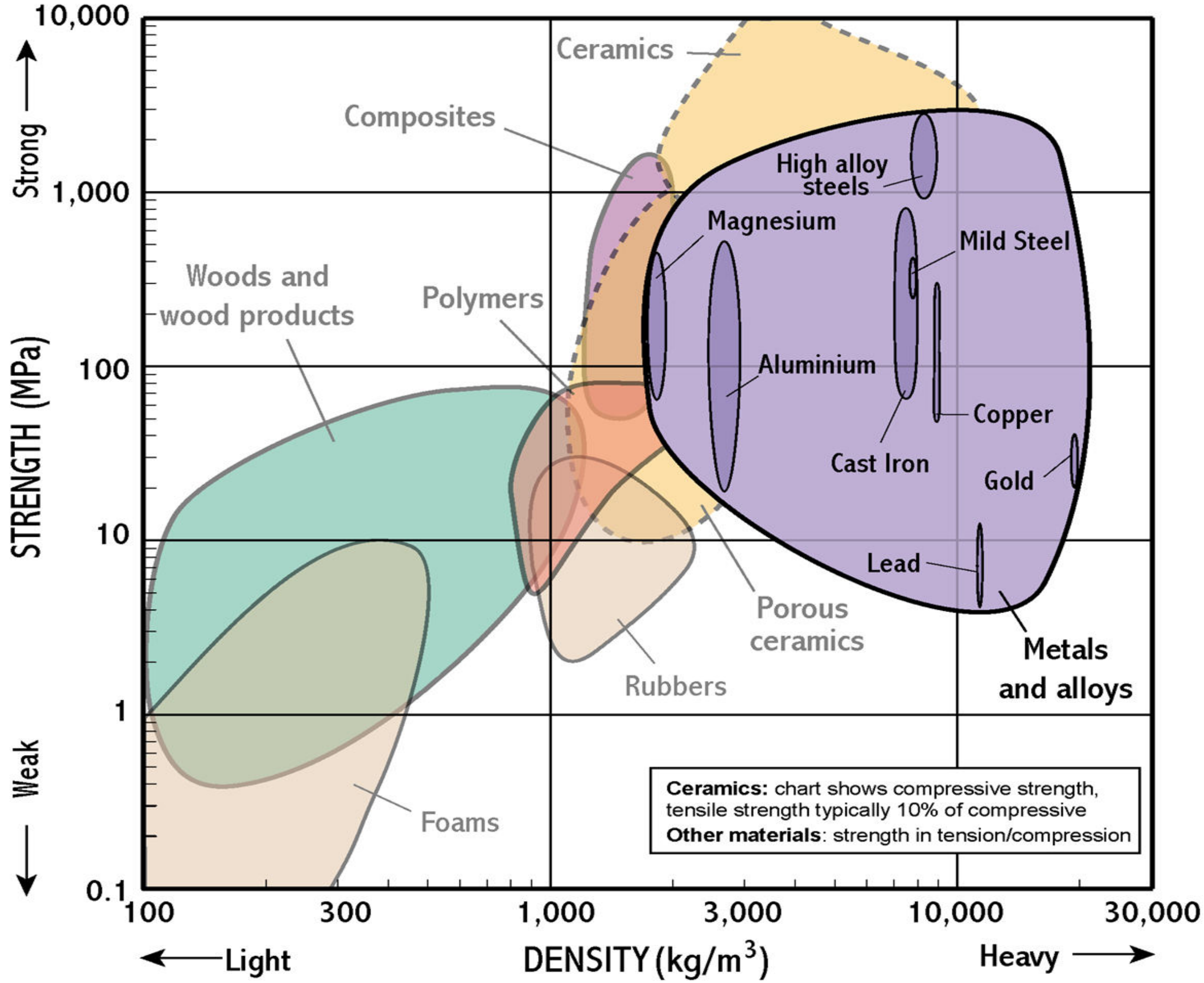

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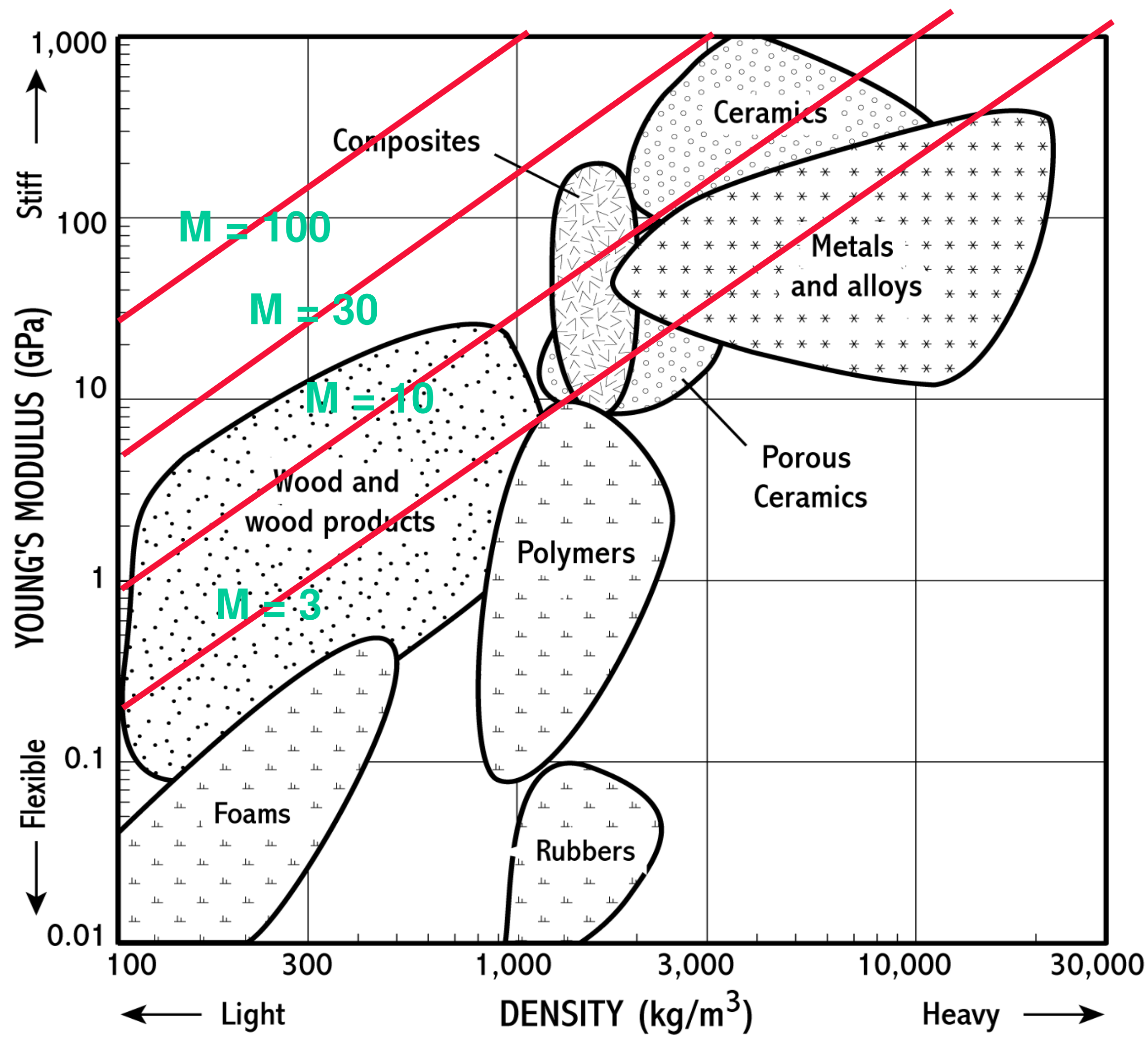
All parallel lines have the same performance

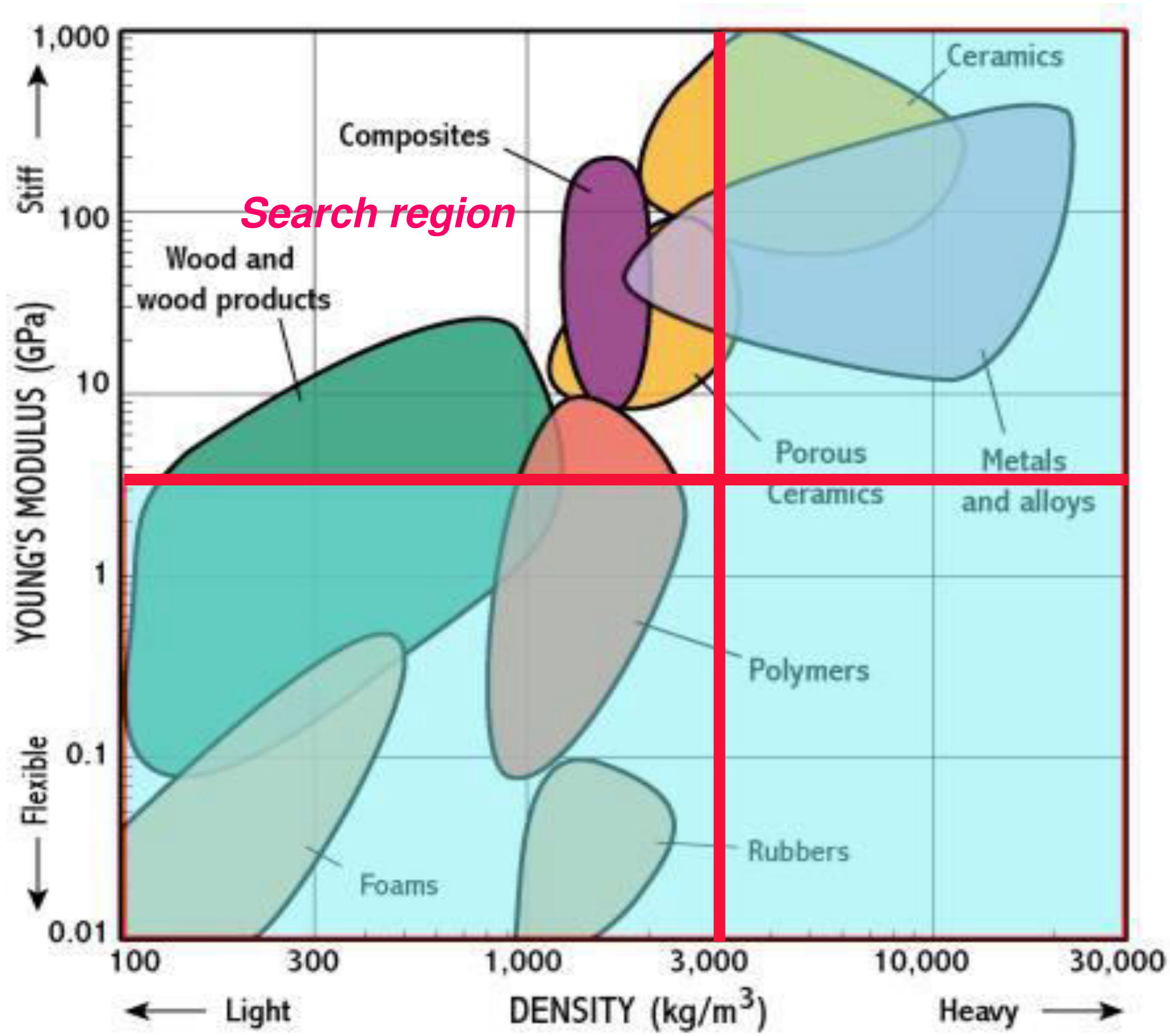
M = 100 is 1/10 the weight of M = 10

M = 30 is 1/3 the weight of M = 10



Constraints on the materials selection charts

- Selection of a material is influenced by constraints.
- Constraints appear as *horizontal* or *vertical lines* on the selection chart
- The primary constraints eliminate blocks of materials leaving a *viable search region*.
- The next step in selecting a material operates only on those materials which are left inside the search region



Red lines are
Constraints

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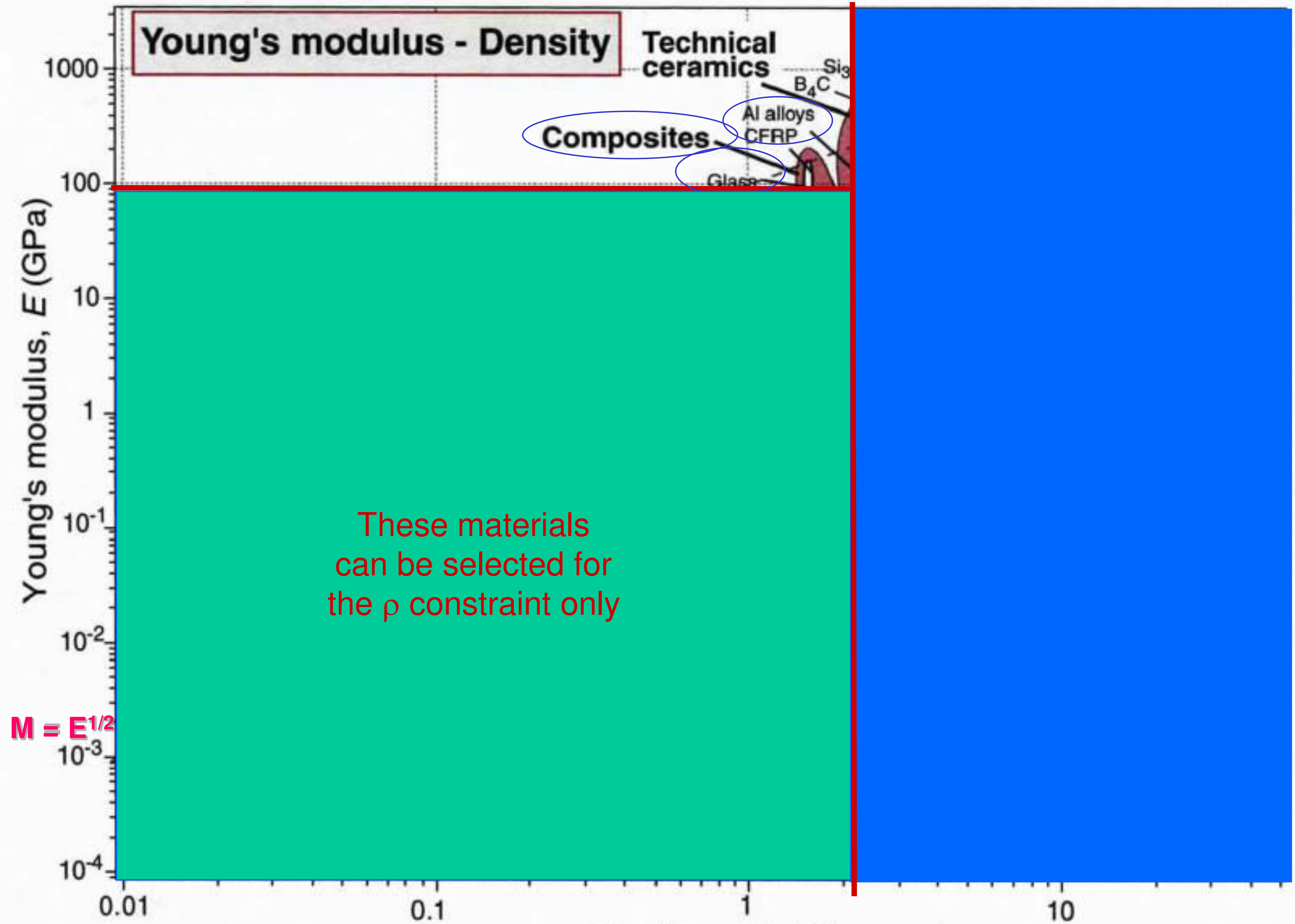
Example

Use the E- ρ chart to find the material with modulus E > 90 GPa and density ρ < 2 Mg/m^3



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Multiple Constraints

- Most materials selection problems are over-constrained with more constraints than free variables.
- e.g; a component may require minimum weight, but with constraints on stiffness, strength and toughness which must also be met.
- This requires the use of several performance indices and several materials selection charts to identify the optimum material

- To solve this problem the following steps are considered:

1. Identify the most important constraint [minimum weight without yielding (plastic deformation)] and identify the appropriate materials performance index. Ignore the remaining constraints.

2. Use this to identify a subset of materials which maximise this performance index.

3. Repeat for the next constraint(s) giving a second or a third performance index.

4. Then, identify a subset of materials which satisfy all performance indices.

- The second index may be plotted on the same chart as the first. The sector isolated above the two lines contains the subset of materials which satisfy the two criteria: $E^{1/2} /\rho > 8\ GPa^{1/2} /Mg/m^3$ and $E > 10$ GPa.

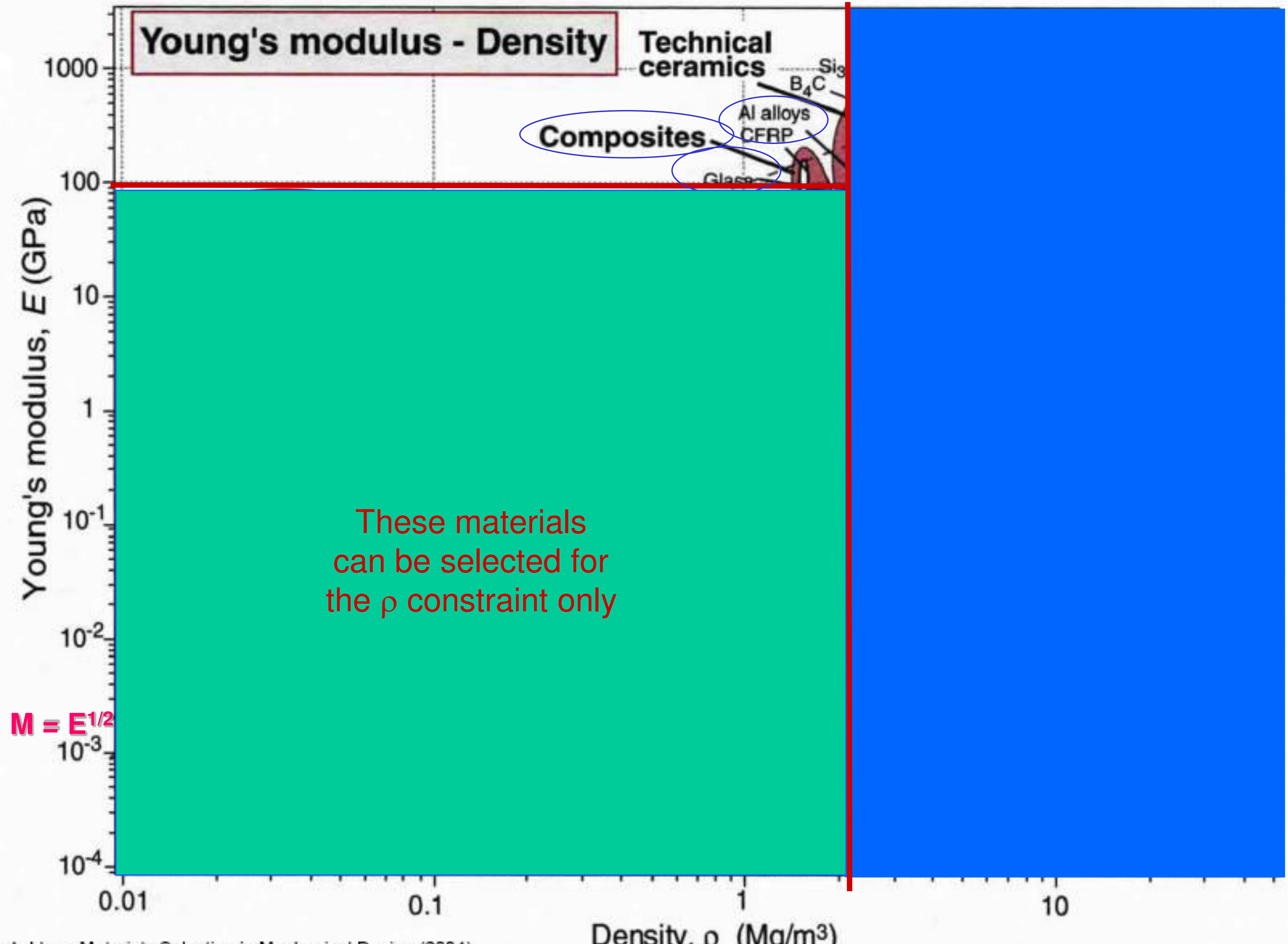
- More often, however, the extra constraint involves a property which does not appear on the first chart.

- In this case, the members of the first subset of materials are tabulated, ranking them using a grid of performance index values

Example

1. Use the E- ρ chart to find the material with modulus $E > 100$ GPa and density $\rho < 2\ Mg/m^3$

2. Use the E- ρ chart to identify the subset of materials with both modulus $E > 100$ GPa and the performance index $M = E^{1/2} / \rho > (6\ GPa)^{1/2} / (Mg/\ m^3)$



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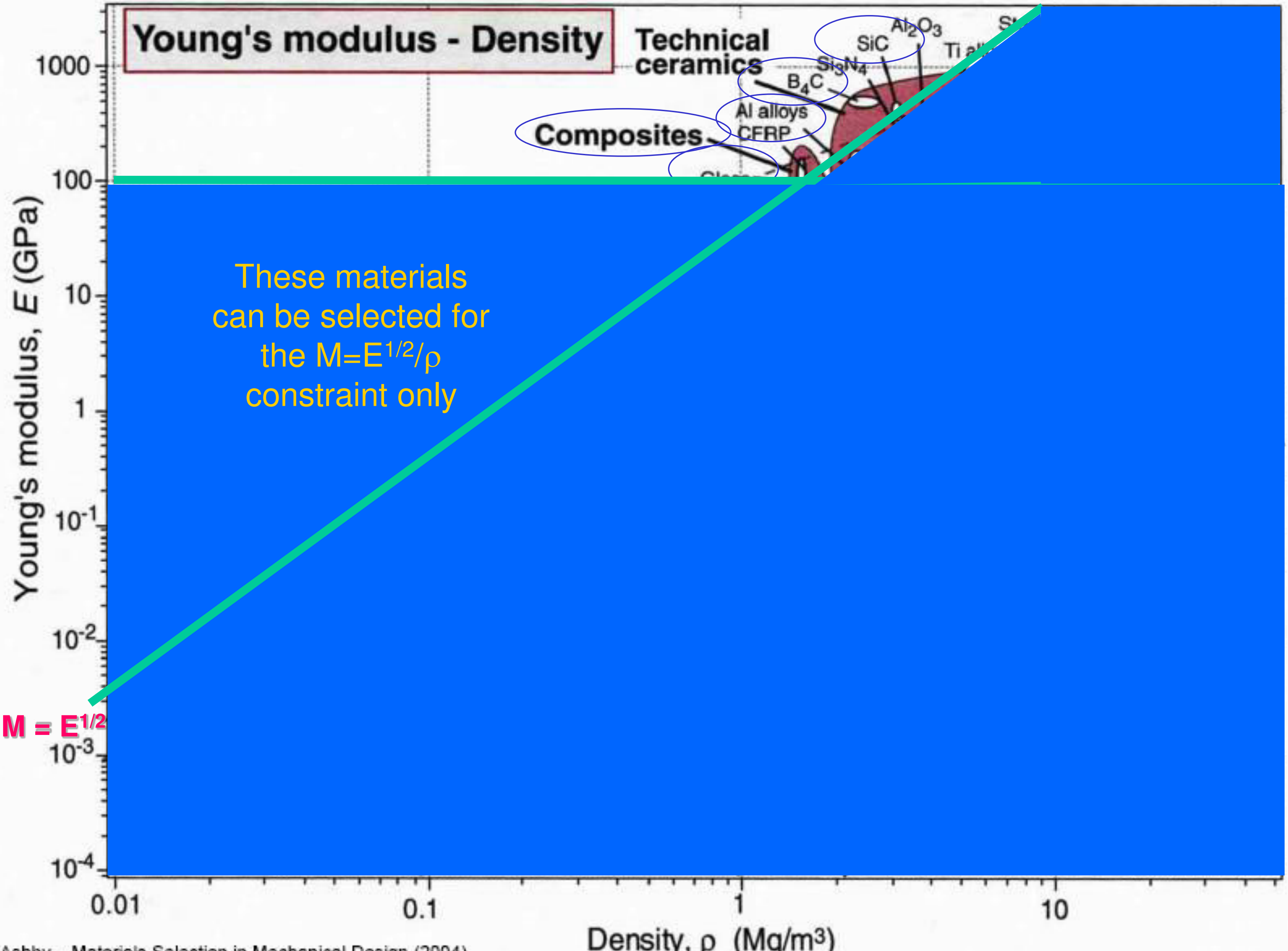

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- The second index is used, with the appropriate chart, to identify a second subset of materials. Common members of the two subsets are identified and ranked according to their success in maximising the two performance indices.

- Consider a problem with one design goal, one free variable and two constraints.

- The result is two equations for the performance, each with the equation:

$$P = f_1\,(F)\,.\,f_2\,(G)\,.\,f_3\,(M)$$

$$P = g_1\,(F)\,.\,g_2\,(G)\,.\,g_3\,(M)$$

- The performance is maximised by choosing:

1. Material with the largest "f_3 (M)"
2. Material with the largest "g_3 (M)"

- The two equations must have equal values. The two performance indices are coupled.

$$\frac{f_3(M)}{g_3(M)} = \frac{g_1(F) \times g_2(G)}{f_1(F) \times f_2(G)}$$

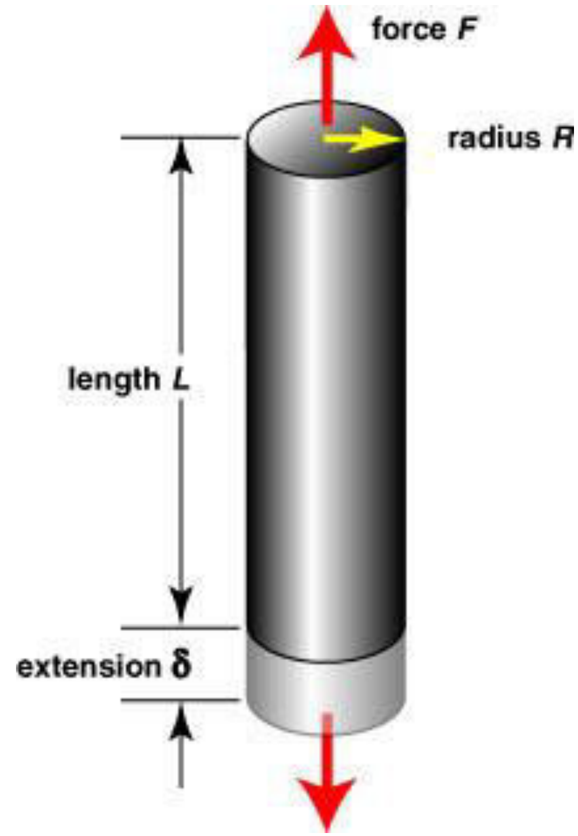
Example 4: Performance Index for a light, stiff and strong tie

- The tie (loaded in tension) is to support a load F, at minimum weight, without failing or extending by more than δ

- *Objective function*: reduce weight

$$m = A\,L\,\rho \qquad (1)$$

- Specified dimensions: *F and L*
- Constraints: failure and stiffness
- Free variable: cross-sectional area, *A.*



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- We need to derive 2 performance indices
- M_1 for the stiffness constraint has already been derived in example 2 as:

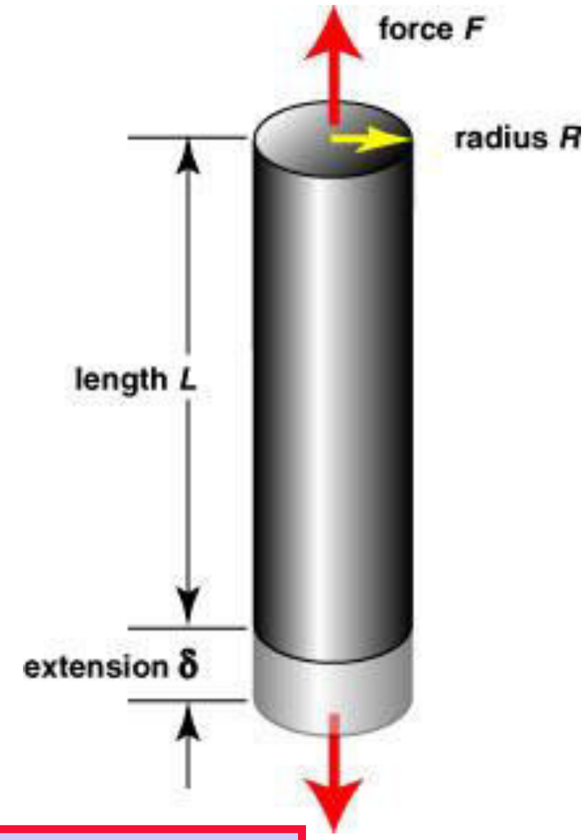
$$M_1 = \frac{E}{\rho}$$

- M_2 for the failure constraint has also been derived in example 2 as:

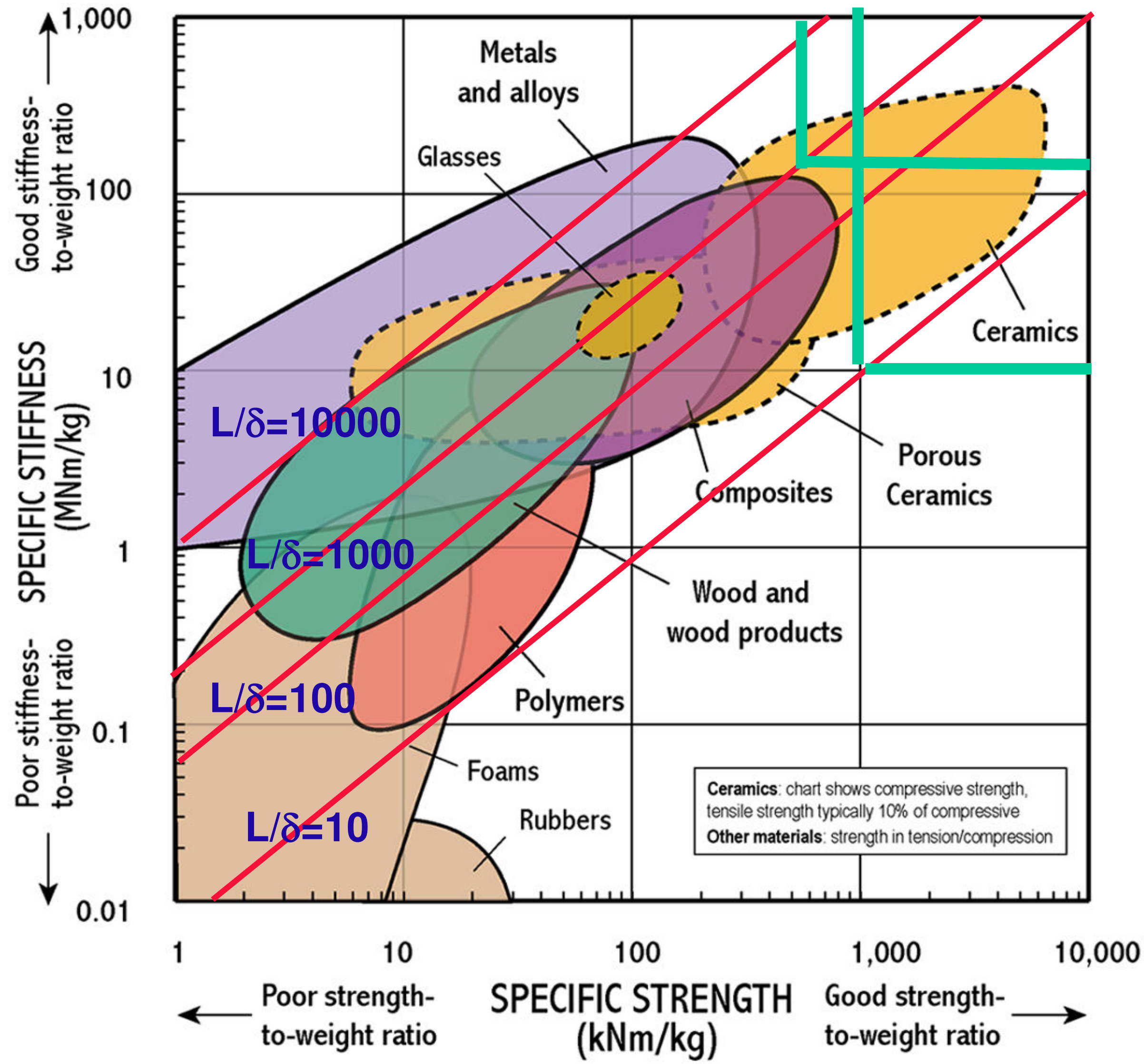
$$M_2 = \frac{\sigma_f}{\rho}$$

- Since the weight is the same so that: $M_1 = M_2$

This is called the "*coupling equation*"



$$\frac{E/\rho}{\sigma_f/\rho} = \frac{L}{\delta}$$



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Multiple Design Goals

- For example, a design goal is to minimise weight, another is to minimise cost (how is weight to be compared to cost (they have different units?)

- The designer must assess the relative importance of all design goals by using "*weighting factors*" to each design goal

- First, the design goals are ranked in order of importance: a numerical factor of "10" is given to the most important goal and a factor of "1" is given to the least important

- The performance index for each design goal (starting with the most important) is determined

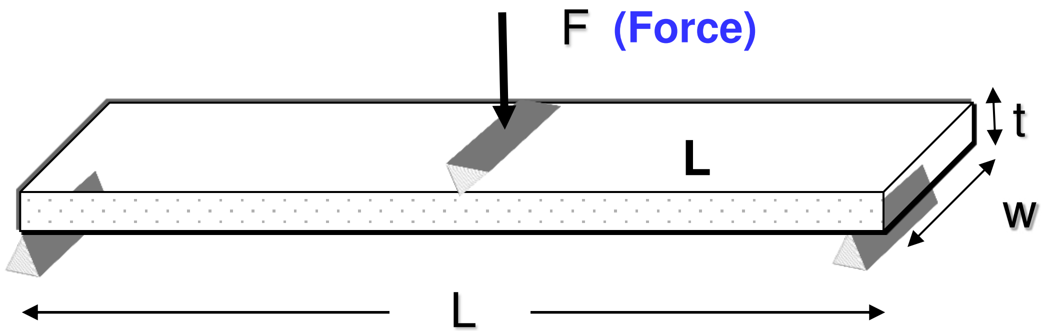
- Finally, the overall performance index is calculated by combining the performance index M obtained from each design goal

- Where $a_1 > a_2 > a_3$, are the weighting factors.

$$M = a_1 M_1 + a_2 M_2 + a_3 M_3 + \ldots$$

- Alternatively a trade-off between the objectives should be used. This is illustrated in the next example.

Example 5: Stiff electronic casing (notebook) with minimum thickness and weight



FUNCTION	Bending
OBJECTIVES	1. Minimum thickness, t 2. Reduce weight: $m = AL\rho = w\,t\,L\,\rho$
CONSTRAINT	Stiffness $S = F/\delta = CEI/L^3$, $I = w\,t^3/12$
FREE VARIABLE	t

See example 3.1.5

OBJECTIVE 1 Minimum thickness, t

Using the constraint equation we get, t, as:

$$t = \left(\frac{12SL^3}{CEw}\right)^{1/3} \rightarrow \boxed{M_1 = E^{1/3}}$$

OBJECTIVE 2 Reduce weight, m

Using m = w t L ρ and replacing t into the mass equation we get:

$$m = AL\rho = \left(\frac{12Sw^2}{C}\right)^{1/3} L^2 \left(\frac{\rho}{E^{1/3}}\right) \rightarrow \boxed{M_2 = \frac{E^{1/3}}{\rho}}$$

- For multiple objectives, we need to determine the relative performance indices.
- That is, suppose the casing is currently made of material M_0

1. The thickness of a casing made from an alternative material M, will be different from that made of material M_0 by the factor:

$$\frac{t}{t_0} = \left(\frac{E_0}{E}\right)^{1/3}$$

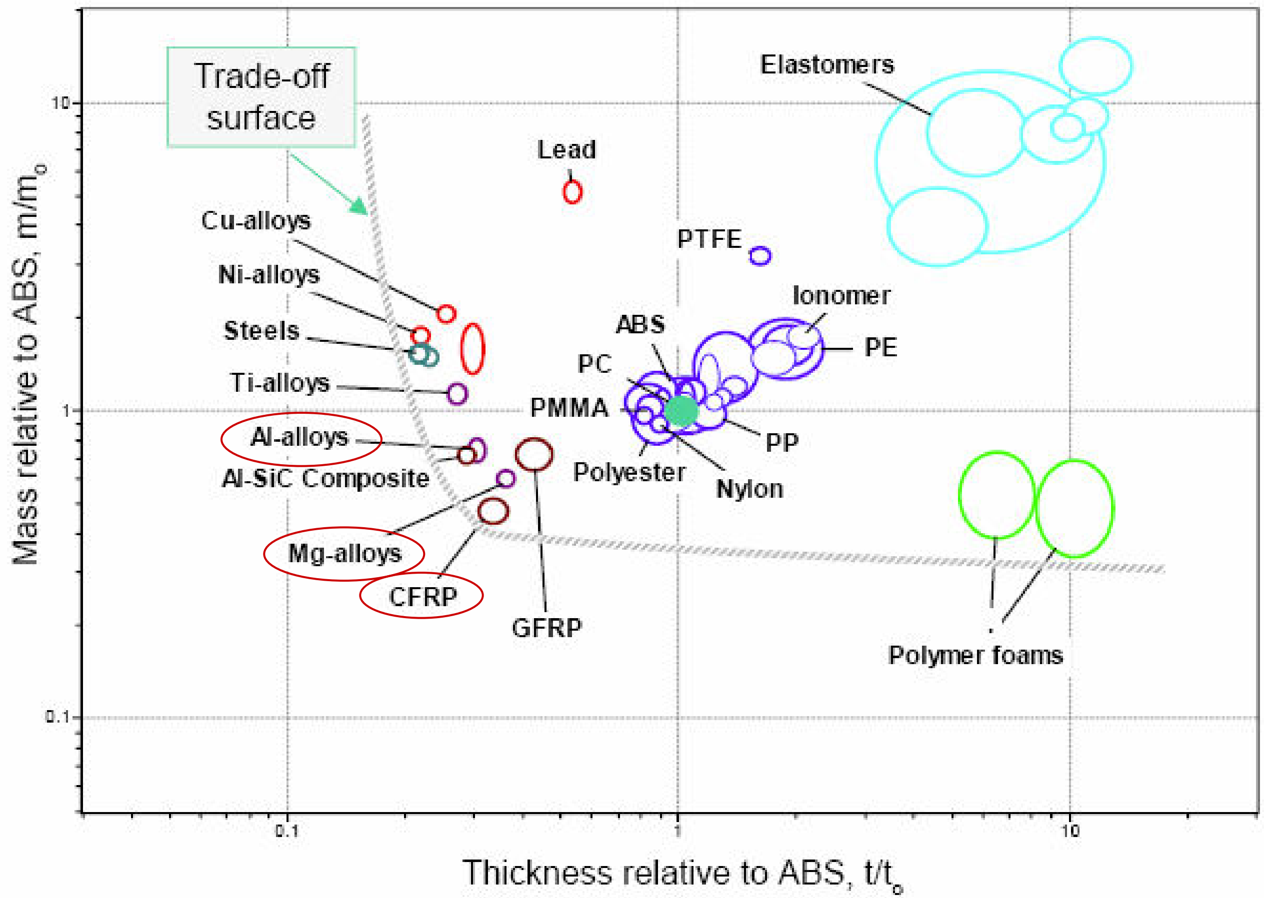
2. The mass of a casing made from an alternative material M, will be different from that made of material M_0 by the factor:

$$\frac{m}{m_0} = \left(\frac{\rho}{E^{1/3}}\right)\left(\frac{E_0^{1/3}}{\rho_0}\right)$$

- The trade-off between thickness and mass can be determined using the chart

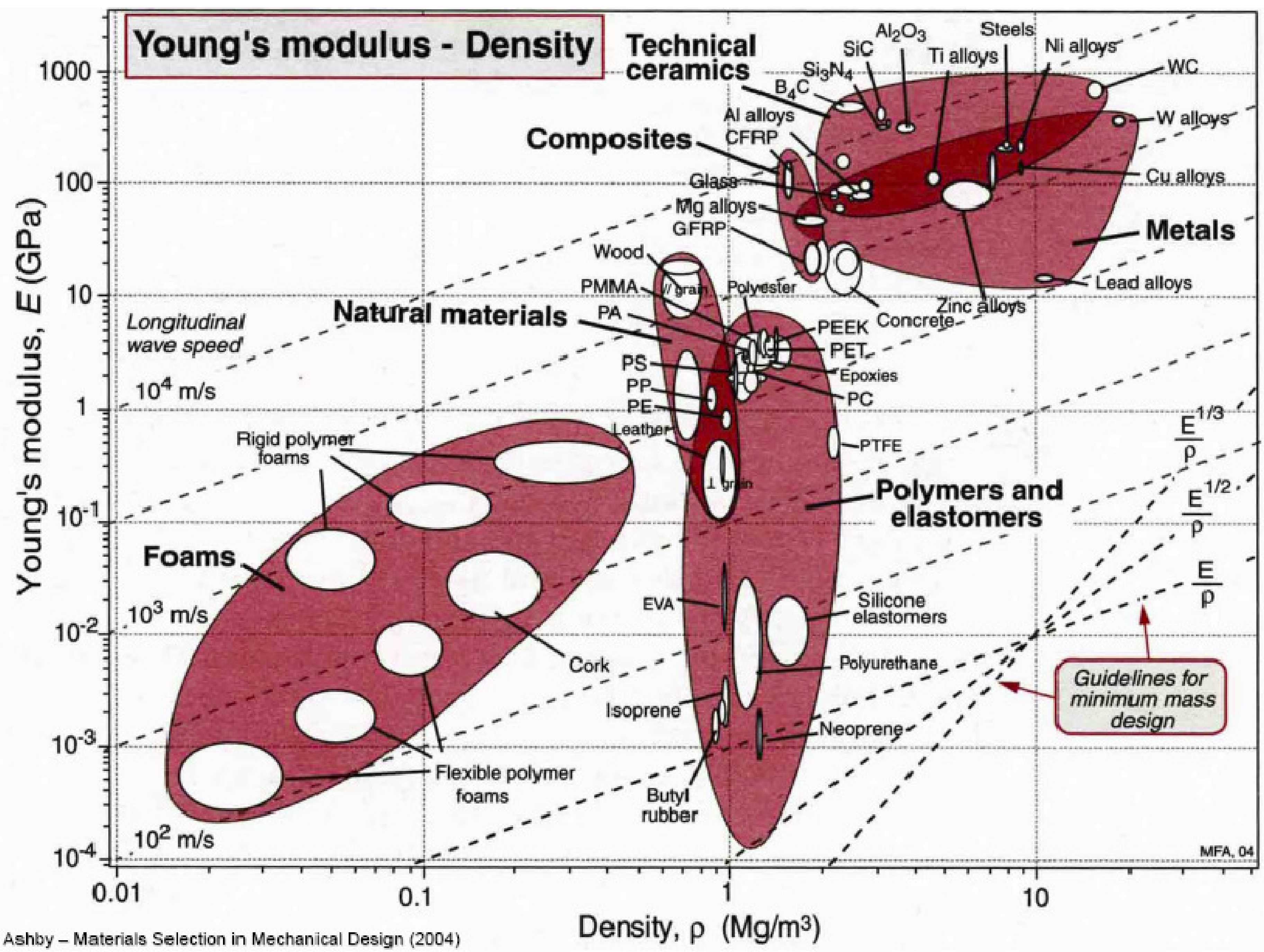
$\frac{t}{t_0}$ and $\frac{m}{m_0}$

TRADE-OFF PLOT



POSSIBLE MATERIALS: CFRP, Al & Mg alloys (Offer low mass at minimum thickness)

MATERIALS SELECTION CHARTS



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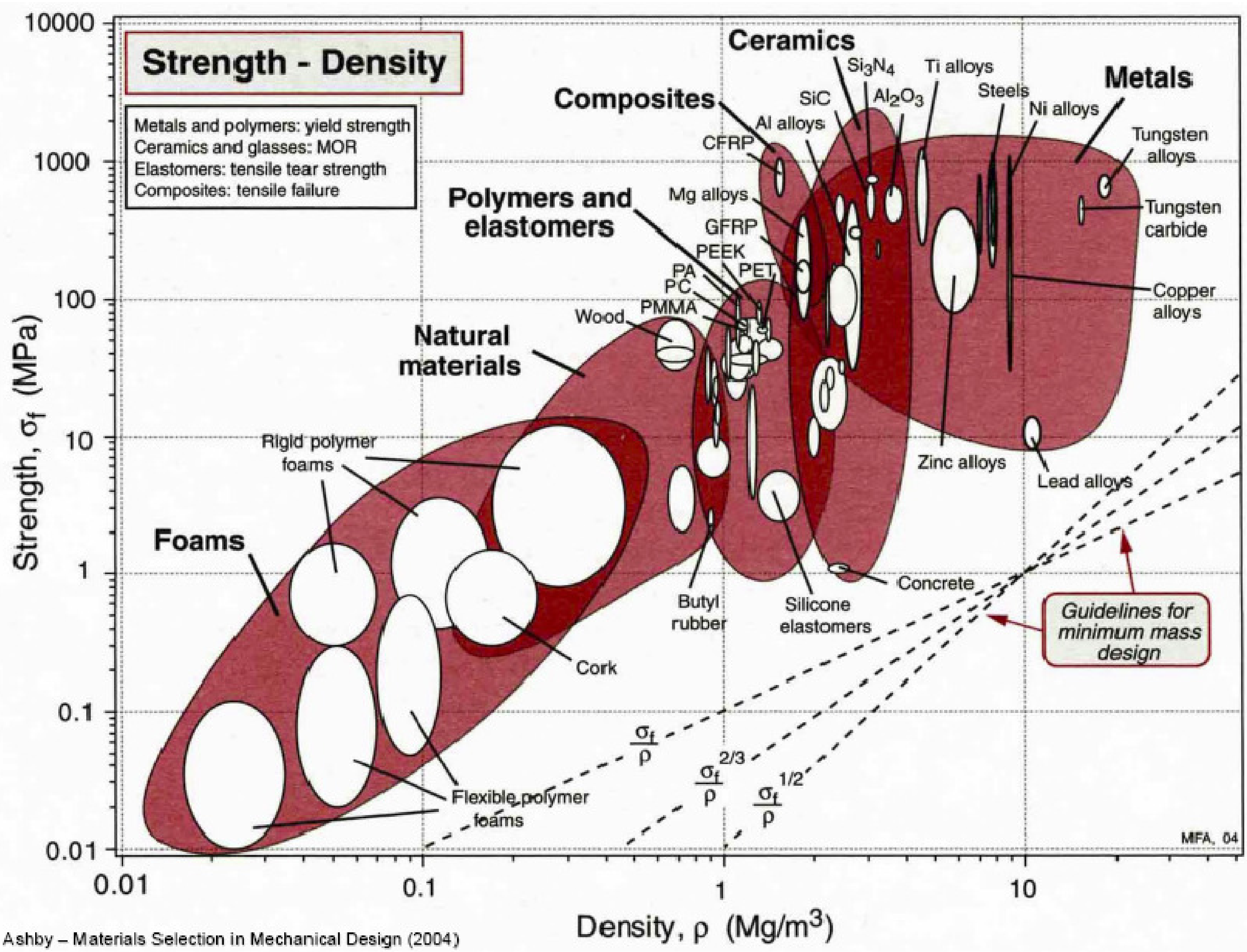

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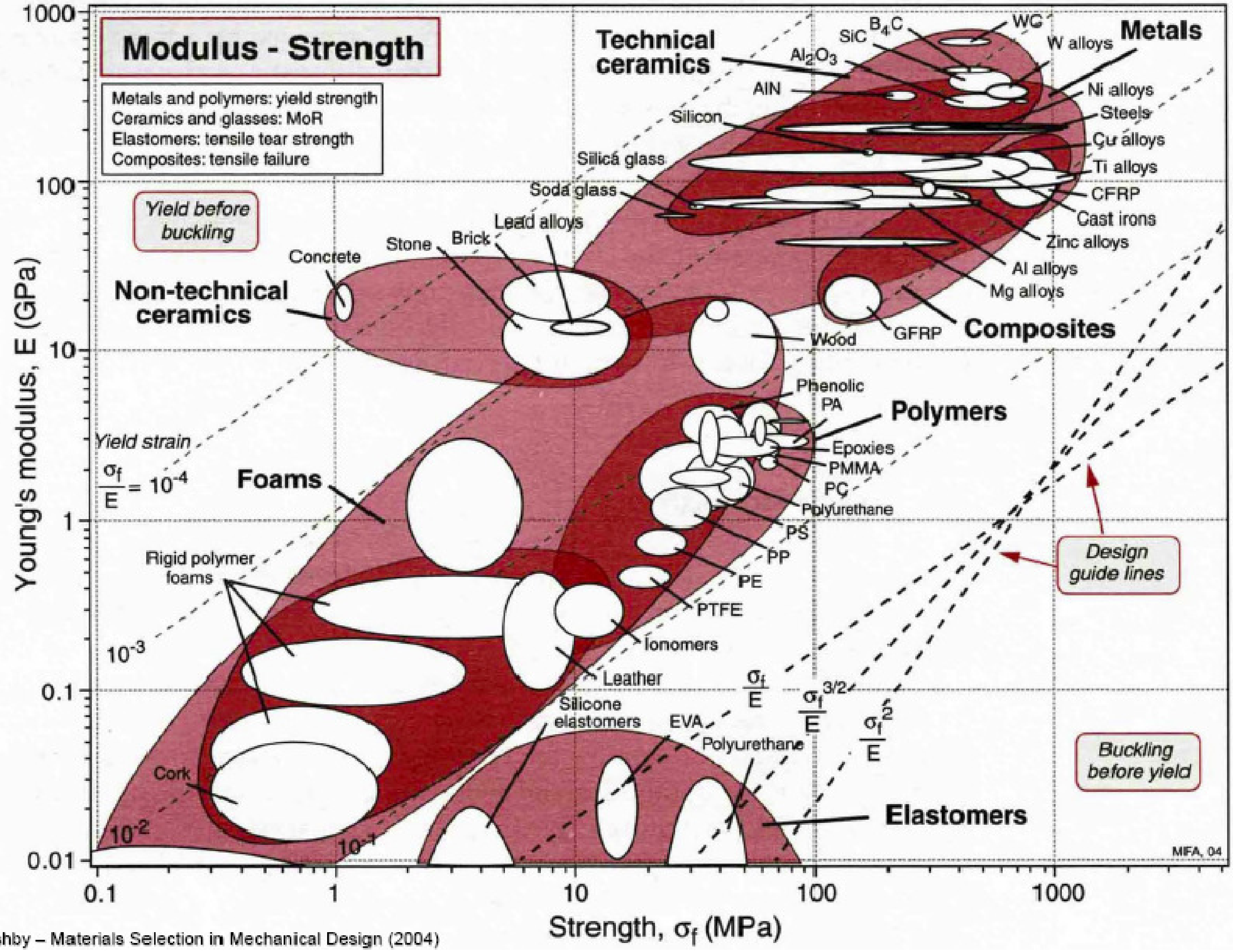

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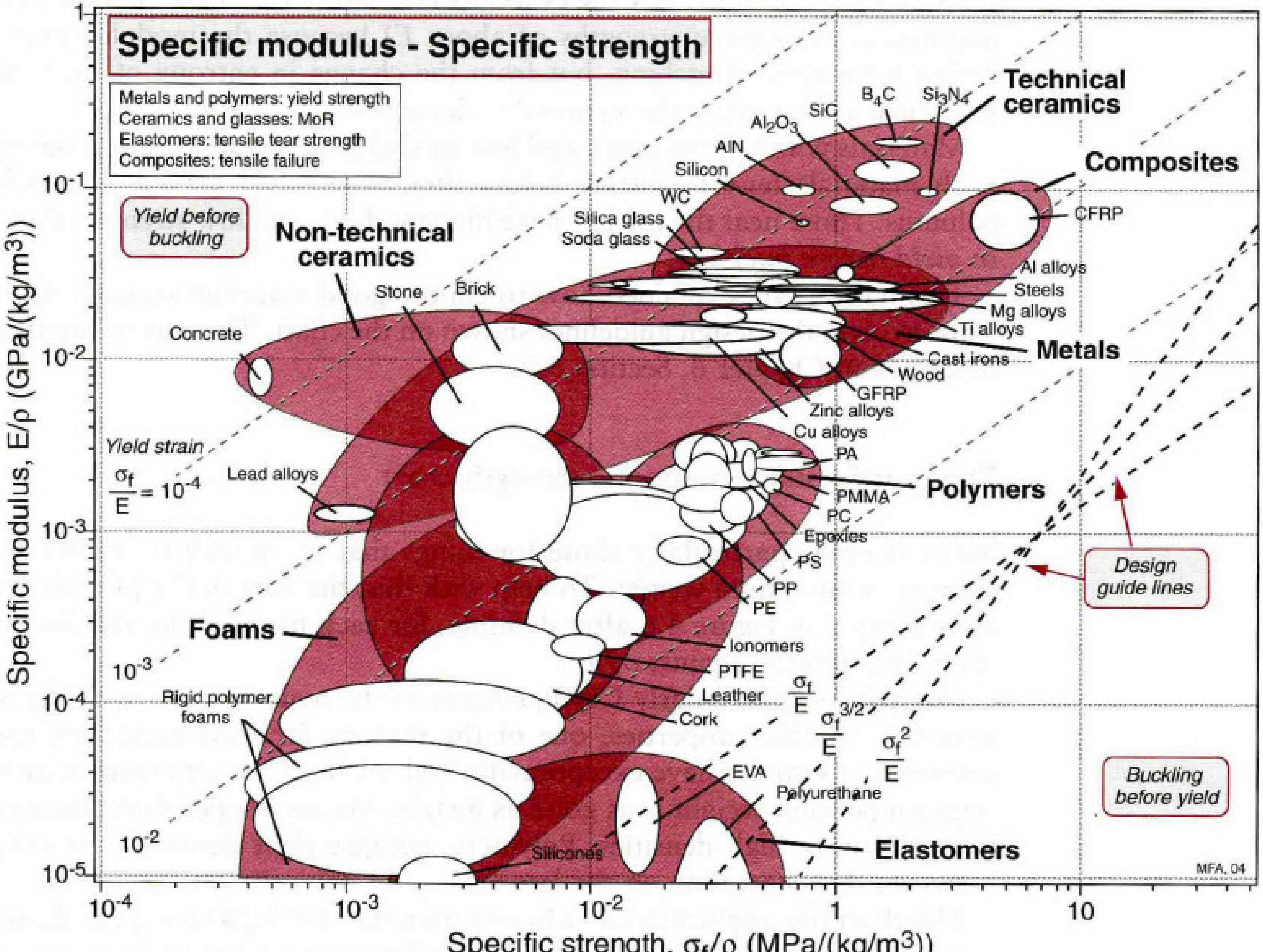

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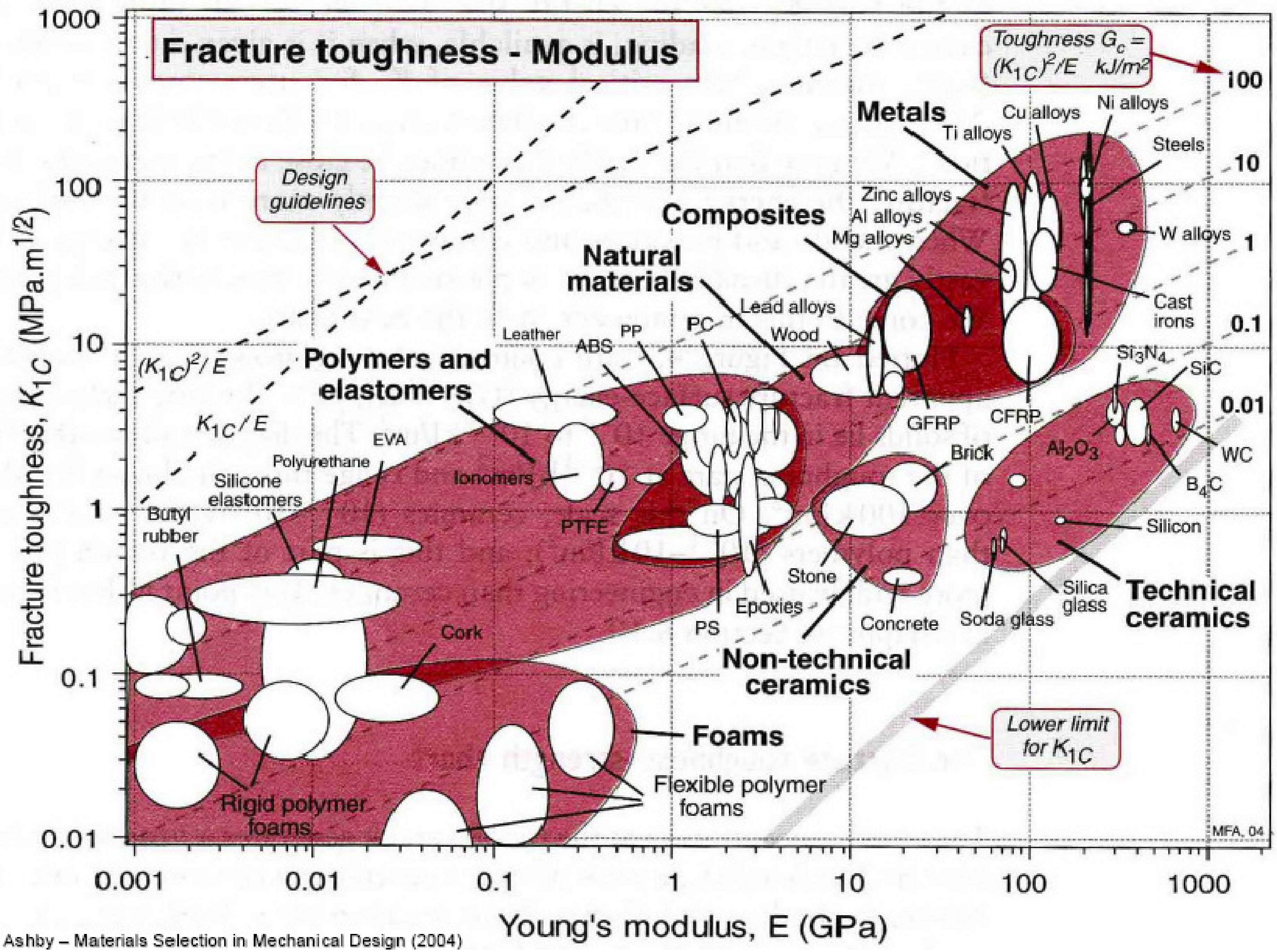

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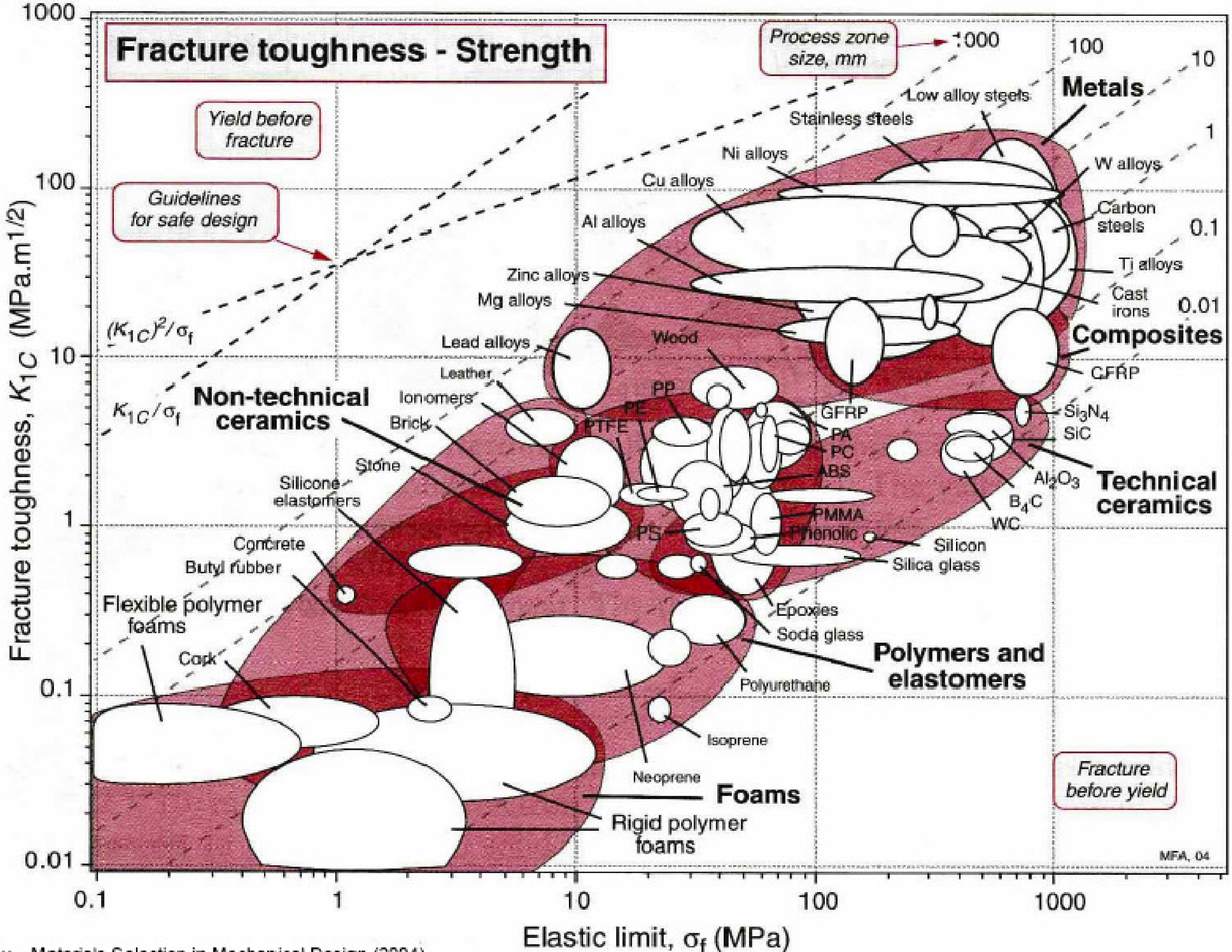

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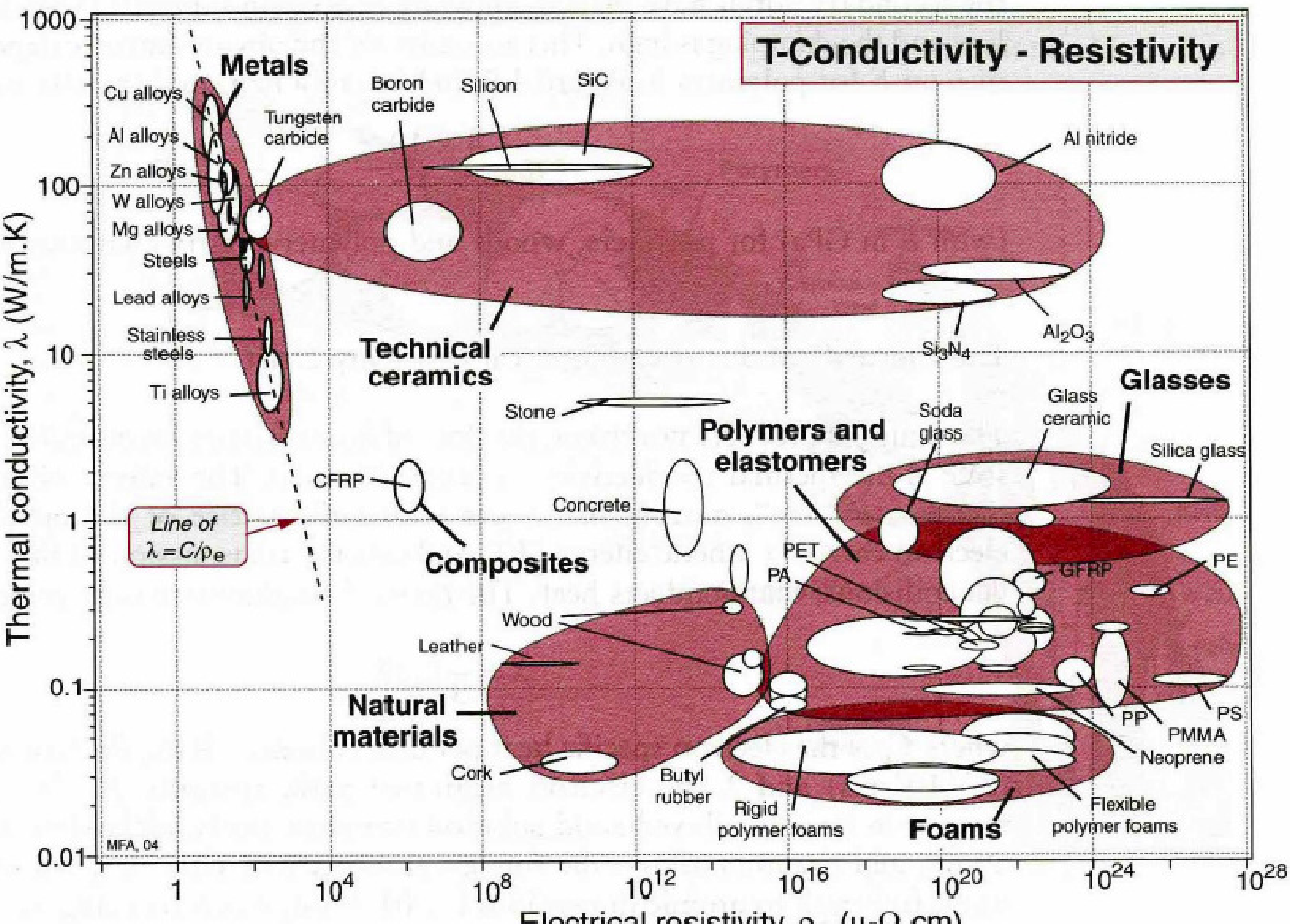

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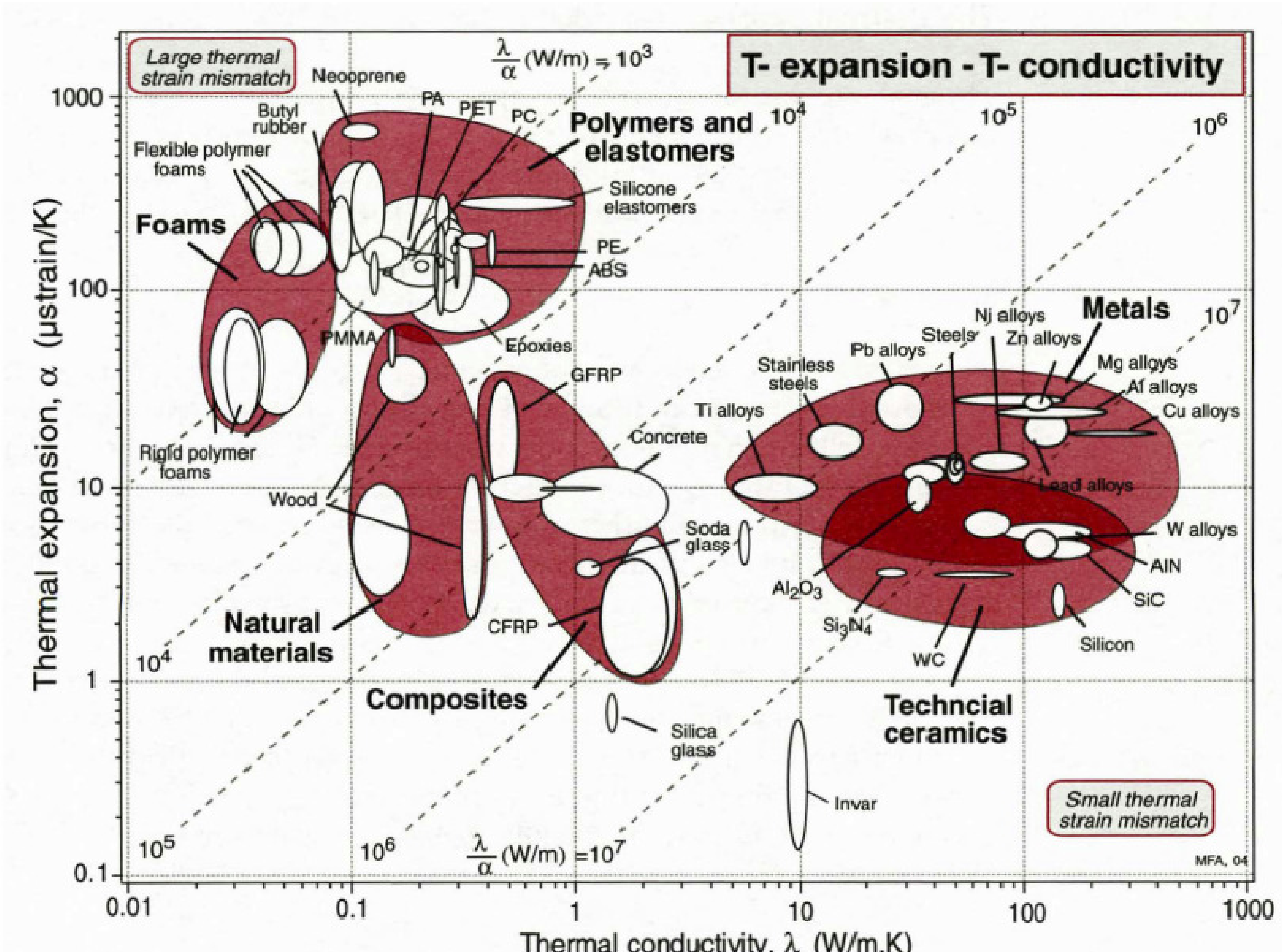

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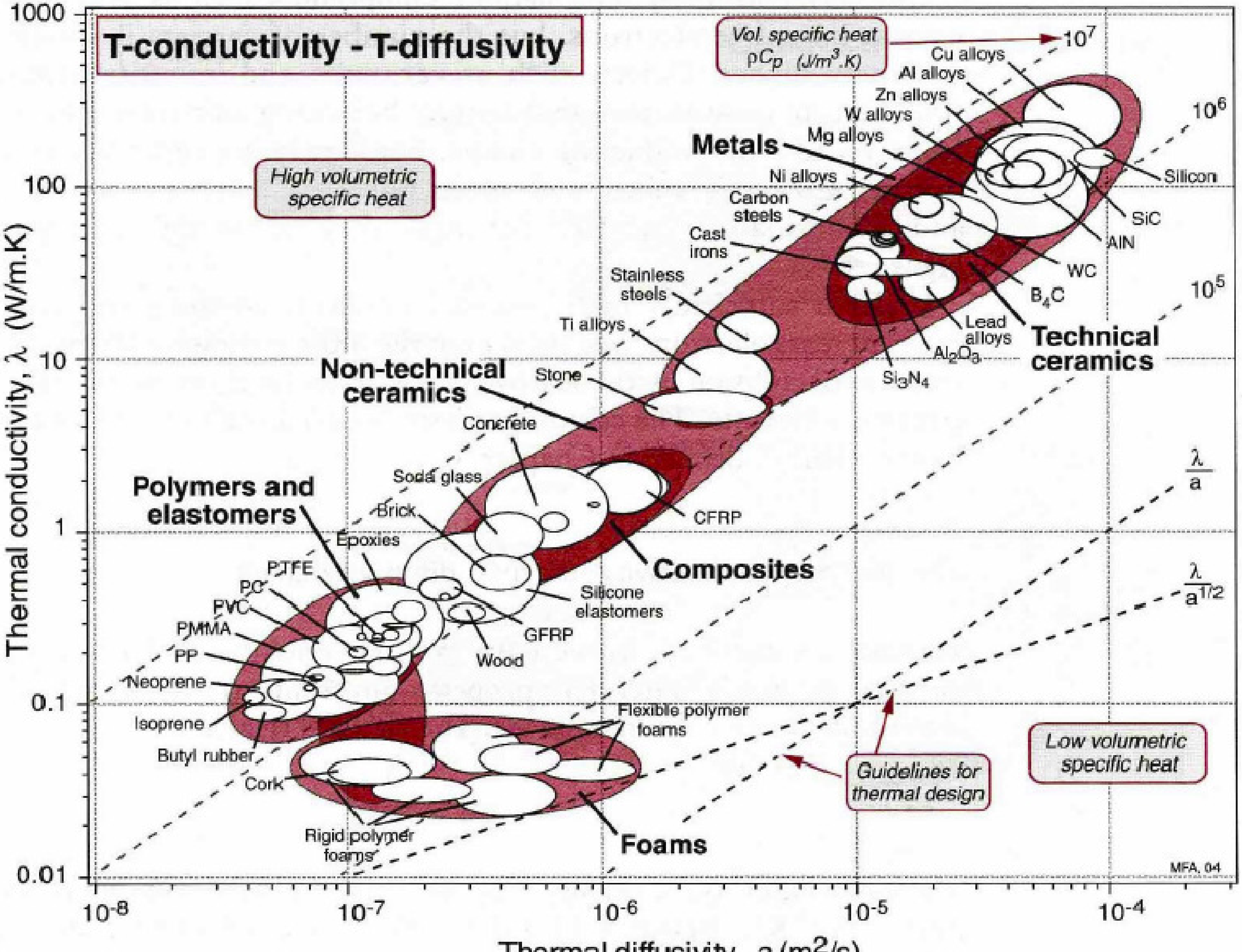

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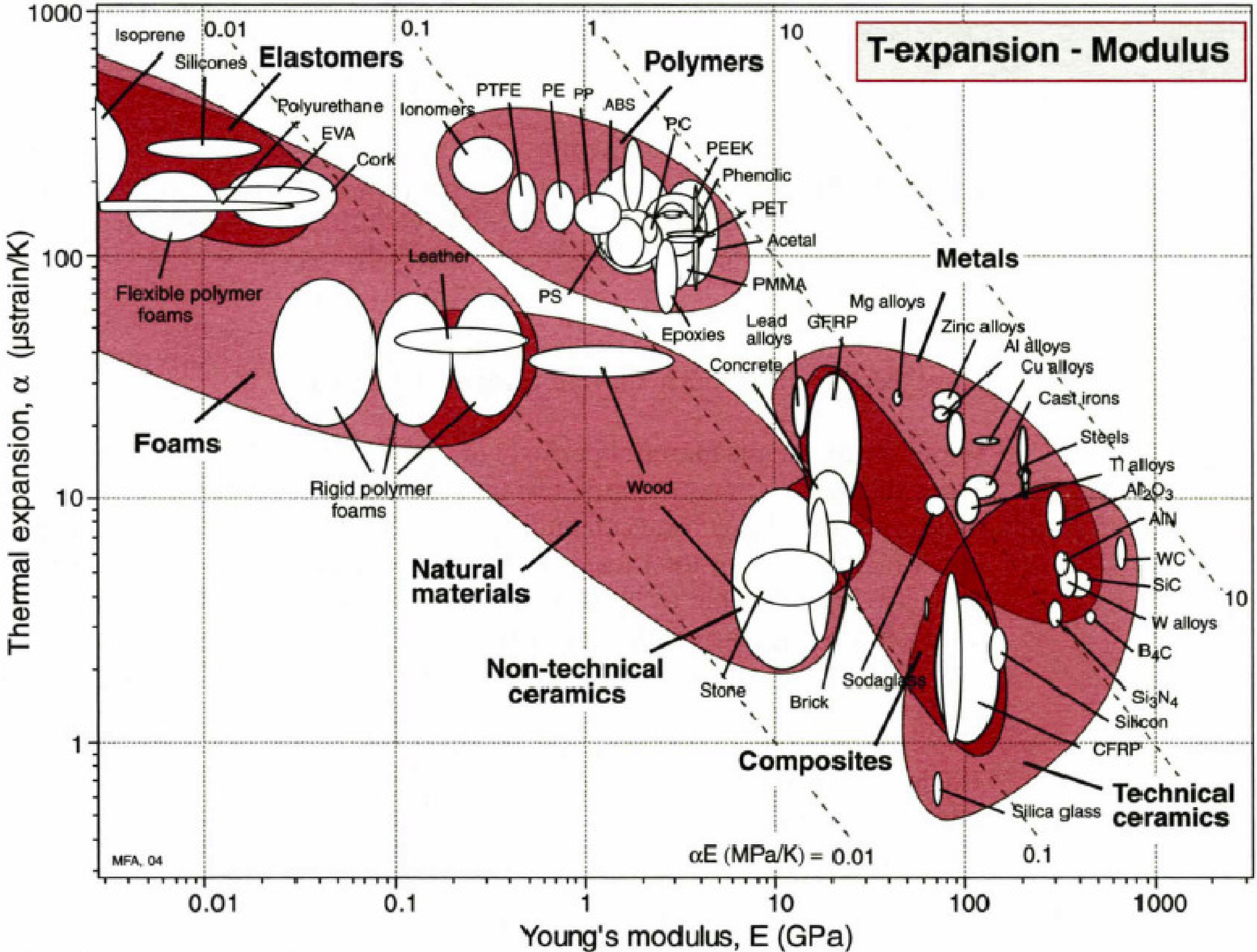

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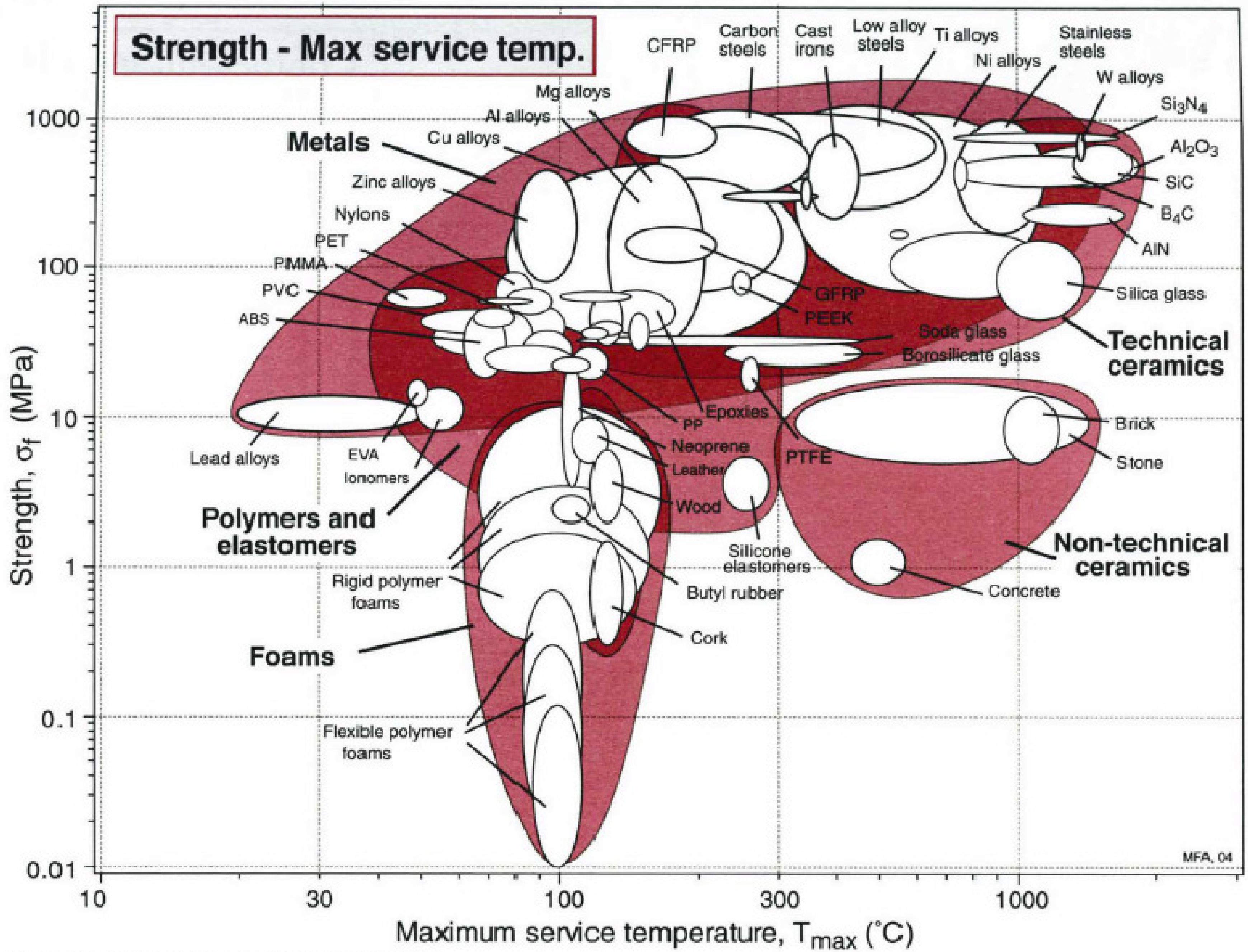
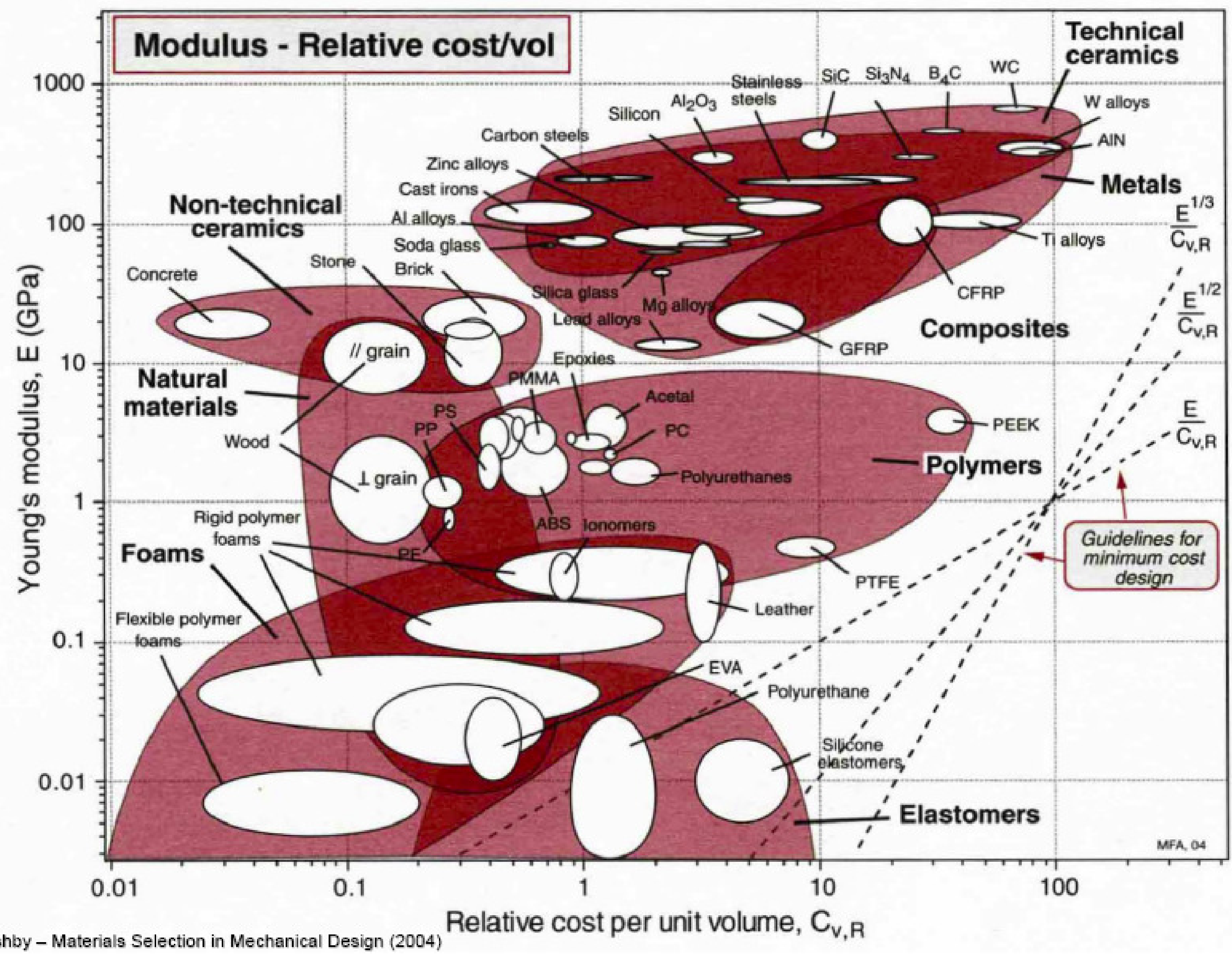


Ashby – Materials Selection in Mechanical Design (2004)



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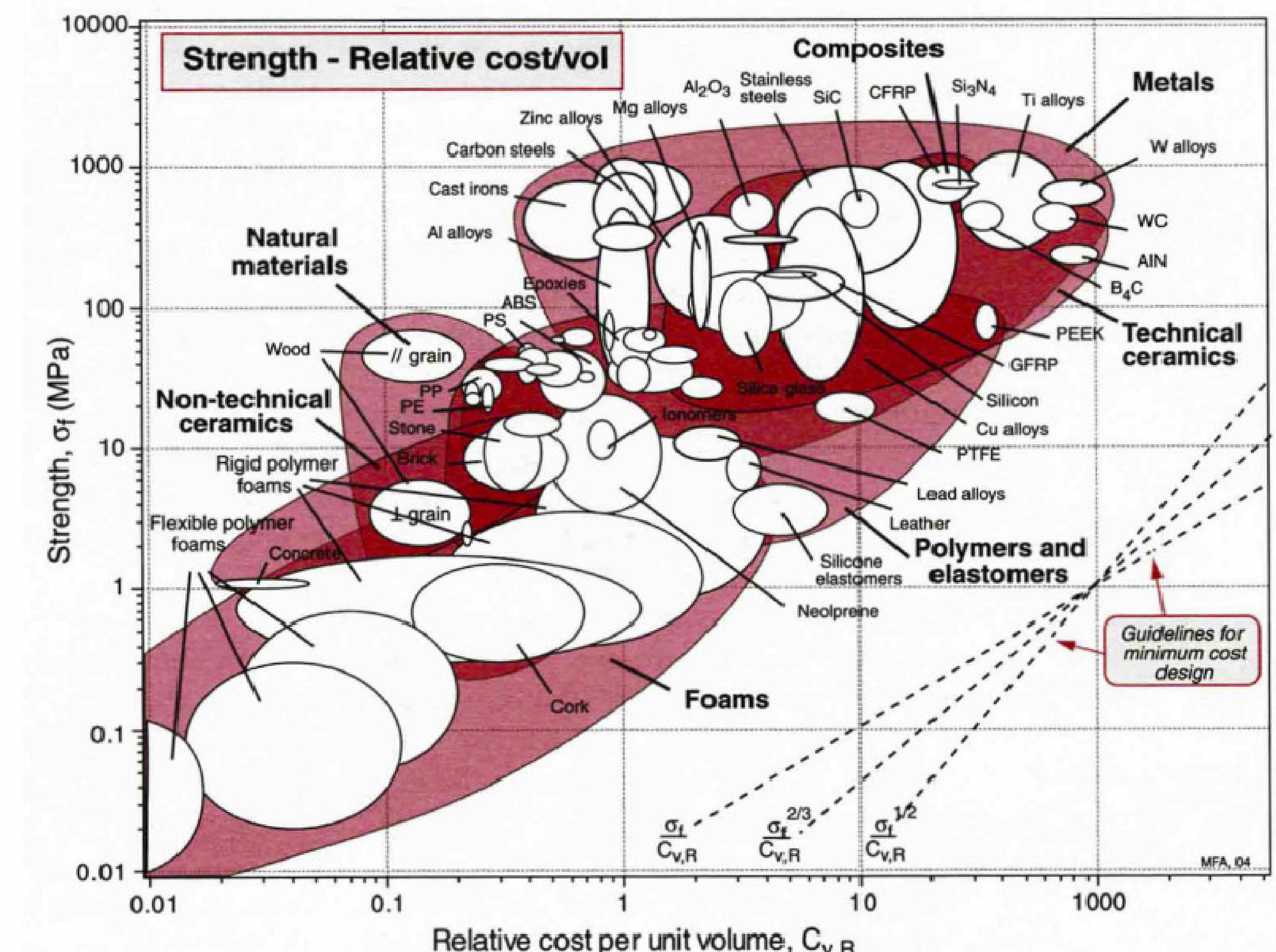

Ashby – Materials Selection in Mechanical Design (2004)